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Illustrating the Possibility of Modelling the Gravity Anomalies in Terms of Bodies Having Density Vary with Depth

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Abstract – *The main focus of this study is to test the hypothesis that the Backus-Gilbert inversion method can be used to model gravity anomalies in terms of bodies having density vary with depth in terms of local averages using constructing averaging kernels and applying it for a special case of a geological fault..*

Keywords: *Backus and Gilbert method, averaging kernels*

INTRODUCTION

Backus and Gilbert (1967, 1968 and 1970) described a mathematically rigorous approach of solving the inverse problem in geophysics for the under-determined case. This inversion method is based on a mathematical abstraction called an "Earth Model". An ordered n-tuple of functions of positions that is sufficient to define an idealized model is known as an "n-dimensional Earth model". Such a model can be considered as a point in an infinite dimensional linear space of all conceivable Earth models. In this method, a model that satisfies a set of observations is created by minimizing the distance between an initial model and the real Earth model in the parameter space, subject to the constraints imposed by observations.

Green (1975) described how the Backus and Gilbert method can be used to model gravity anomalies in terms of constant density bodies. This paper investigates the possibility of using this method to model gravity anomalies in terms of bodies with increasing density applying it for a special case of a geological fault.

A normal geological fault is formed as a result of breaking of a large Earth structure into two blocks and subsiding one of them relative to the other. The void created by the subsidence of the block will normally be filled with sediments and density contrast between the sediments (ρ_s) and the other block that did not subside (ρ_c) produces a gravity anomaly. It is possible to model a normal fault by merely modelling the region of sedimentary rocks as a semi-infinite slab with a density contrast of $(\rho_s - \rho_c)$. In such a body, the sediments in the lower part of the sequence are consolidated and have higher densities while those on the upper part are not consolidated and therefore have lower densities. It will be shown here that it is possible to recover this vertical density variation using the Backus and Gilbert method.

This study is further extended to test the hypothesis that the Backus-Gilbert inversion method can be used to model gravity anomalies in terms of bodies having density vary with depth in terms of local averages using constructing averaging kernels.

As explained by Meneke (1984) it is necessary to have an infinite amount of information in the form of observations to predict the continuous variation of a certain physical property of the Earth such as density or electrical conductivity with depth. In real world studies only a limited amount of data or observations are available. In such instances instead of predicting the continuous variation of the physical quantity with depth it is possible to predict how the local average of the physical quantity over a certain interval around a given point vary with depth using the Backus and Gilbert method. The theory related to determination of local averages by constructing averaging kernels has been explained under Averaging kernel sub heading.

METHODOLOGY

The feasibility of modelling a normal geological fault with increasing densities with depth using the Backus and Gilbert method was examined by inverting the artificially simulated gravity anomaly data due to a hypothetical geological fault of known dimensions and a known density distribution using this method and then comparing the results obtained with the relevant features of the hypothetical fault.

Figure 1 shows the hypothetical fault with density that increases with depth and the gravity anomaly it produces used for the numerical experiment. First the anomaly was modelled in terms of a body of constant density using the version of the Backus and Gilbert method for gravity modelling presented by Green (1975) after suitably modifying it for modelling of a fault using the following steps.

1. Divide the initial trial 2D of rectangular cross sectional shape into a number of semi-infinite slabs.

2. Determine density contrast of each slab using the Backus and Gilbert method together with the singular value decomposition method.

3. Neglect the semi-infinite slabs having positive density values or density values close to zero by taking their thickness equal to zero.

4. Assign a particular value for the density of remaining semi-infinite slabs and calculate the gravity anomaly due to the new body and compare it with the "Observed anomaly". If the agreement is not acceptable, then repeat steps 2 and 3 above until a suitable agreement is obtained.

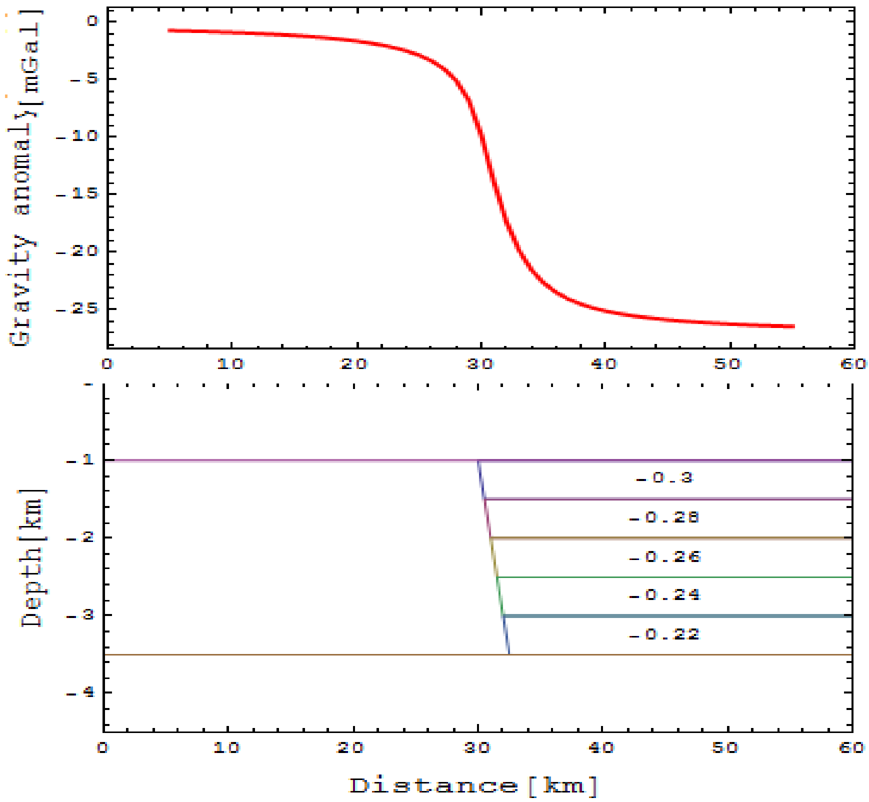

Figure 1: The shape of the body (collection of 2D semi-infinite slabs) and the gravity anomaly it produces.

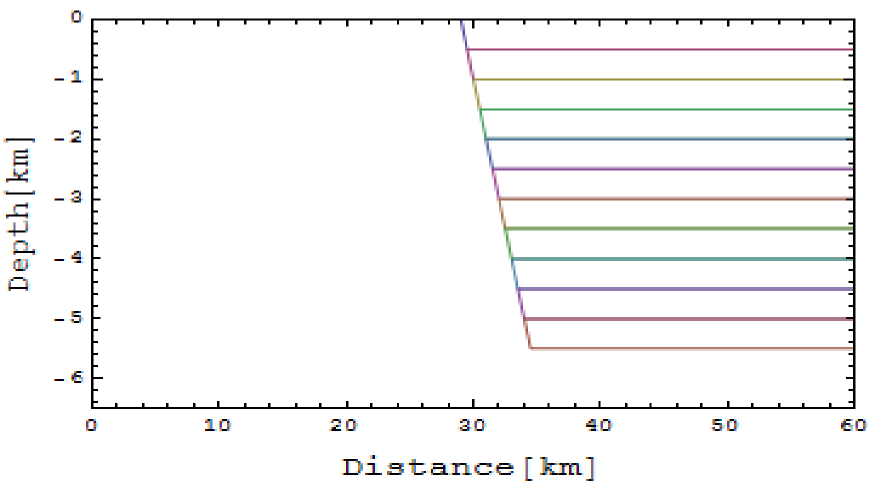

Figure 2: The shape of the initial trial body of the iteration process.

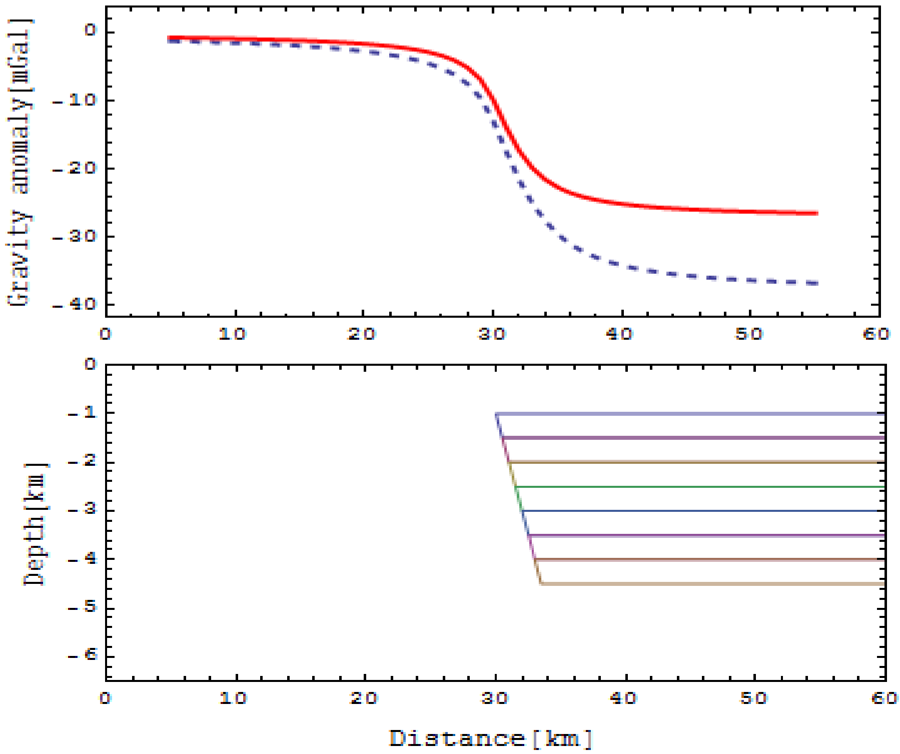

Figure 3: Gravity model obtained after the first iteration and the gravity anomaly it produces (blue dashed line).

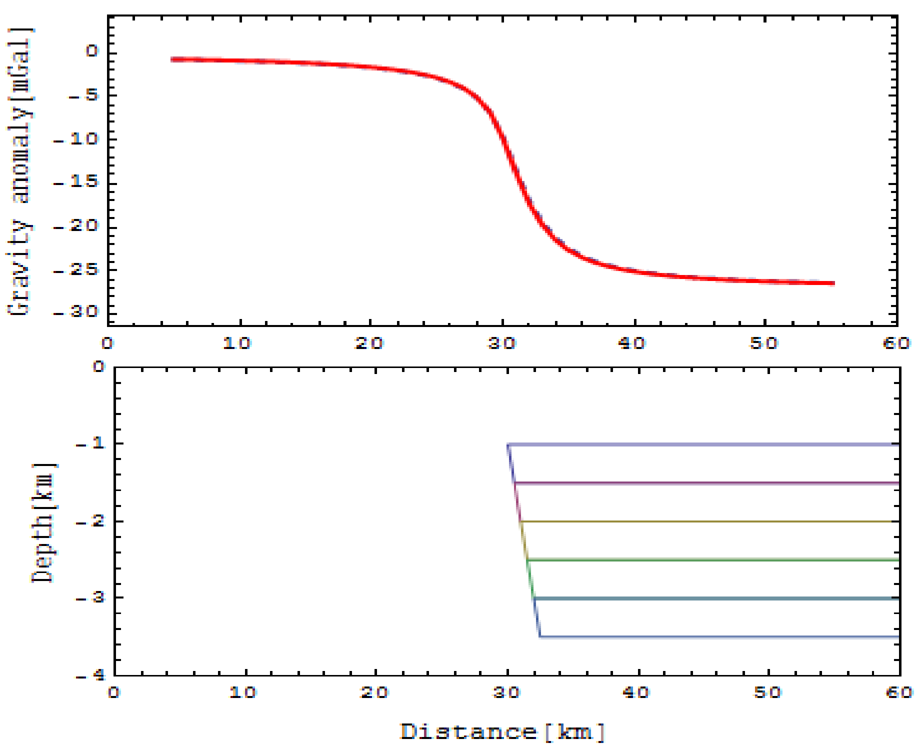

Figure 4: Gravity model obtained after the fifth iteration and the gravity anomaly it produces (blue dashed line).

Gravity anomaly due to a two dimensional model fault (Figure 1) has been computed using the "Wolfram Mathematica 7". The model obtained by the use of the above method consisted of five consecutive slabs of equal density contrast of -0.3 g/cm3. Then to model a body with increasing densities, each slab of the model was divided into five horizontal strips so that the whole model is now divided into 25 strips. It is well known that in shallow depths, density of sediments increases linearly with depth. A density value for each strip was assigned a value assuming a linear increase of density with depth. Then using the Backus and Gilbert method, an attempt has been made to determine a density distribution for strips that is closest to the assumed values and also satisfies the observations.

For the test example, depth to the basement is considered as 3.5 km while its density contrast values are -0.3, -0.28, -0.26, -0.24, -0.22 g/cm^3. Gravity anomaly was calculated over a range of 50 km at intervals of 1km. The averaging kernels of each layer were calculated using the "Wolfram Mathematica 7".

Density values obtained for the each strip of each slab and the constant density value used for the each slab for the hypothetical fault are given in the Table 1 and also illustrated in Figure 5. As can be seen from this Table 1, densities of the set of first five strips are very closely equal to the density of the first slab of the hypothetical fault. Similarly the densities of 2nd, , 3rd, 4th and 5th sets of five strips have densities approximately equal to the densities of 2nd,3rd, 4th and 5th slabs of the hypothetical fault.

Singular value decomposition

The Singular value decomposition (SVD) is one of the most versatile and useful tool in Linear Algebra. Linear algebra is the study of linear operators which map vectors between two vector spaces. Matrices are convenient representations of linear operators.

The purpose of singular value decomposition is to reduce a dataset containing a large number of values to a dataset containing significantly fewer values, but which still contains a large fraction of the variability present in the original data. Often in the atmospheric and geophysical sciences, data will exhibit large spatial correlations. SVD analysis results in a more compact representation of these correlations. This technique is a generalization of the eigenvector decomposition of matrices to non-square matrices and is explained below.

In the SVD method a data kernel, an $N \times M$ matrix G is factored into
$G = U\,S\,V^T$ so that,
U is an $N \times N$ orthogonal matrix with columns that are unit basis vectors spanning the data space, R^N.
V is an $M \times M$ orthogonal matrix with columns that are basis vectors spanning the model space, R^M.
S is an $N \times M$ diagonal matrix with diagonal elements called singular values.
The singular values along the diagonal of S are customarily arranged in decreasing size, $s_1 \geq s_2 \geq s_3 \geq \cdots\ s_{min\,(N,M)} \geq 0$. Note that some of the singular values may be zero. If only the first p singular values are nonzero, we can partition S as

$$S = \begin{bmatrix} S_p & 0 \\ 0 & 0 \end{bmatrix}$$

where S_p is a $p \times p$ diagonal matrix composed of the positive singular values. Expanding the SVD representation of G in terms of the columns of U and V gives

$$G = [u_1, u_2, u_3, \dots\ u_N] \begin{bmatrix} S_p & 0 \\ 0 & 0 \end{bmatrix} [v_1, v_2, v_3, \dots\ v_M]^T$$

$$G = [U_p \quad U_0] \begin{bmatrix} S_p & 0 \\ 0 & 0 \end{bmatrix} [V_p \quad V_0]^T \tag{1}$$

where U_p denotes the first p columns of U, U_0 denotes the last $N-p$ columns of U, V_p denotes the first p columns of V, and V_0 denotes the last $M-p$ columns of V. Because the last $N-p$ columns of U and the last $M-p$ columns of V in [1] are multiplied by zeros in S, we can simplify the SVD of G into its compact form $G = U_p\, S_p\, {V_p}^T$. Further by using the SVD, generalized inverse of G (G^{-g}) can be obtained as;

$$G^{-g} = V_p {S_p}^{-1} {U_p}^T \tag{2}$$

Averaging Kernel

Backus and Gilbert (1968, 1970) have developed a powerful technique for making reliable generalizations from an incomplete data set. Only a simplified version of their technique will be needed here. Suppose we wish to estimate some function of radius in the Earth $m(r)$ from a set of observations g_i which are functionally dependent, in a known way, on $m(r)$. For a spherically layered earth, we may write

$$\sum_{j=1}^{M} G_{i\,j}\, m_j = g_i \qquad i = 1,2, \dots, N \tag{3}$$

where
$G_{i\,j}$ are some known coefficients,
g_i are the data and
m_j are the values of the unknown in the M concentric layers.
If $M > N$, the system of equations [3] is underdetermined, and if any solution exists, there is an infinite number of solutions will exist.
Let us sum both sides of [3] over a set of coefficients a_i^k

$$\sum_{i=1}^{N} \sum_{j=1}^{M} a_i^k\, G_{i\,j}\, m_j = \sum_{i=1}^{N} g_i\, a_i^k \tag{4}$$

Supposing that a_i^k can be found such that

$$\text{Let } A_j^k = \sum_{i=1}^{N} a_i^k\, G_{i\,j} \tag{5}$$

is about 1 for $j = k$, and near zero otherwise. Then the right side of equation [4] will have physical meaning as a "local average" of the property $m(r)$ near the k^{th} layer. We may pick a separate set of coefficients a_i for each layer k which we wish to investigate.
We then have

$$\widehat{m}^k = \sum_{i=1}^{N} a_i^k\, g_i \tag{6}$$

as the local average of $m(r)$ near layer k. Note that equation [4] $w$ill be true for any set a_i^k but will have its useful physical meaning only if a_i^k are chosen such that equation [4] has the "delta-like" property advertized.

Table 1: The calculated density values of each semi-infinite slab

	1st Slab (g/cm3)	2ndSlab (g/cm3)	3rd Slab (g/cm3)	4th Slab (g/cm3)	5th Slab (g/cm3)
Assumed	-0.3	-0.28	-0.26	-0.24	0.22
1st Strip	-0.291	-0.277	-0.262	-0.238	-0.219
2ndStrip	-0.289	-0.275	-0.260	-0.237	-0.218
3rd Strip	-0.288	-0.268	-0.247	-0.231	-0.218
4thStrip	-0.278	-0.266	-0.248	-0.230	-0.220
5thStrip	-0.277	-0.262	-0.238	-0.230	-0.212

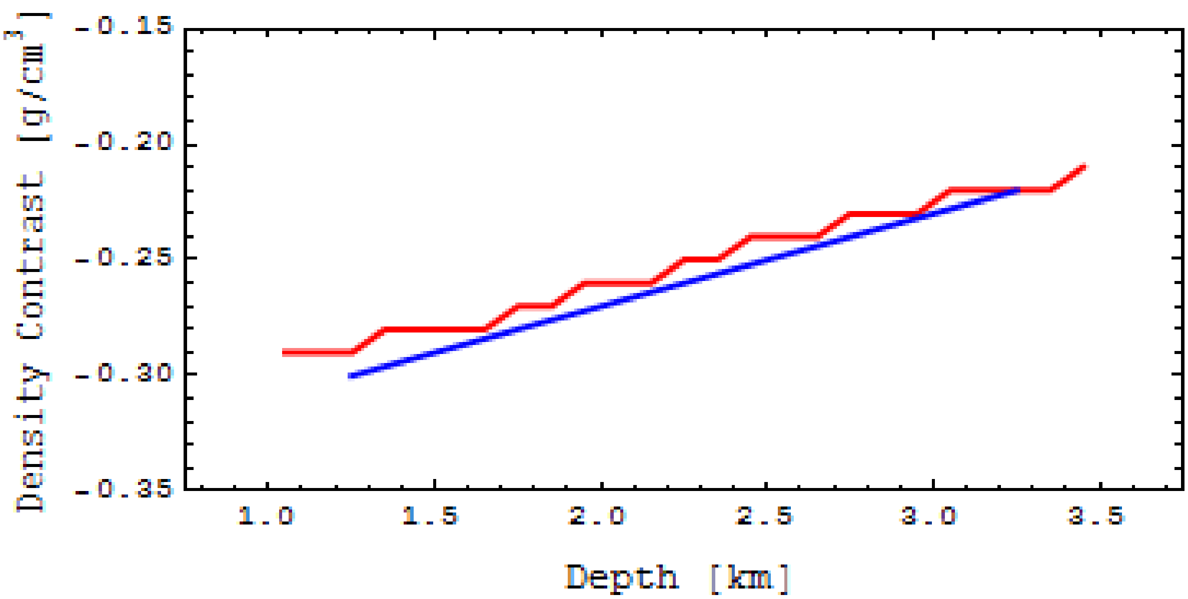

Figure 5: The density contrast values assumed for 5 slabs of the hypothetical model (Straight line)and those calculated for the 25 layers using the Backus and Gilbert method (zigzag line)

DISCUSSION

Even though in most geophysical studies it is assumed that subsurface structures causing gravity anomalies have constant densities, in reality they have densities that increases with depth. Modelling of gravity anomalies using the powerful Backus and Gilbert method was so far limited to modelling of bodies with constant densities. This study illustrates that the Backus and Gilbert method can be extended to model gravity anomalies due to bodies with increasing density. However, the present study is only illustrates the possibility of modelling of gravity anomalies due to a geological fault. The method can be extended for other geological structures such as sedimentary basins and igneous intrusions with appropriate modifications.

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