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Study of Variable Amplitude in Cyclic Loading

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Abstract – *Fatigue failure often initiates from a point of stress concentration, where the local deformation is inelastic. Fatigue is produced by the repeated application of cyclic loads and prediction of fatigue failure is important at the design stage of components to minimize the risk of failure. Therefore, the analysis of elastic-plastic deformation and predict the behavior of the component is an essential for fatigue durability assessment of engineering components. Due to the lack of valid analytical methods to perform an accurate stress analyses, many engineers rely on finite element methods to evaluate elastic-plastic stress/strains in complex components. Many commercially available finite element software use simple flow rules and commonly known yield criteria for the stress/strain calculations. In this study, the finite element method was used to analyze a notched specimen made of mild steel which was subjected to cyclic tensile and torsional loading. The results obtained for strain were compared using a Neuber method.*

The comparison concludes that strain values resulted from Neuber method are 36% higher than the that from Finite Analysis method. This infers that Neuber evaluation method over estimates the results from FEA method (1). However, for an exact validation of the FEA model under realistic conditions further experimental analysis is needed.

***Keywords**: Cyclic Loading, Finite Element Analysis, Neuber Equation*

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1 INTRODUCTION

Finite Element Analysis (FEA) is a computer-based numerical technique for calculating the strength and behaviour of engineering structures. It can be used to calculate deflection, stress, vibration, buckling behaviour and many other phenomena. It can be used to analyze either small or large-scale deflection under loading or applied displacement. Finite element method can be used to analyze both elastic deformation and plastic deformation.

Mild steel is used in a wide range of industrial applications that involve cyclic loading. It is used mainly used for bridges and for manufactureing bolts and fasteners as it has required strength and ductility. These applications may involve multiaxial loading, which can be more damaging for the mateial than the uniaxial condition, especially for nonproportional loading. Hence, an understanding of the cyclic stress-strain response and of the fatigue behaviour under multiaxial loading of these materials is important for safer engineering design.

Finite Element Analysis enables detail evaluation of complex structures in a computer, at the designing stage of these structures. This study investigated the deformations of mild steel having Elastic modulus of 2.08E05 MPa, Poisson ratio, 0.3 and modulus of rigidity, 80 GPa under cyclic loading. The specimen modelling and finite element analysis were carried out by using ABAQUS Code. The new cycle counting method, the Stress Scale Factor (SSF) virtual cycle counting, is based on the SSF equivalent shear stress early proposed by the Benham et al (1996).

2 SAMPLE DESIGN AND FINITE ELEMENT ANALYSIS

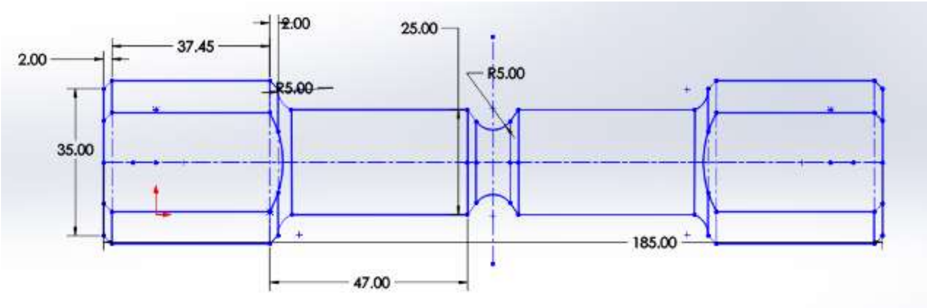

Fig. 1: Dimensions of the specimen

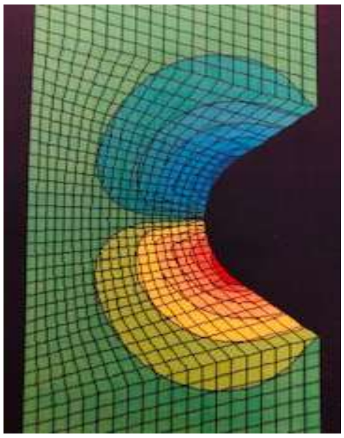

Fig. 2 3D view and the finite element model
(a) 3D view of the specimen (b) Abacus model

Fig. 1 shows the dimensions of the notched specimen and figure 2a and 2b, the 3D view and the finite element model of the specimen. The specimen was modelled by using ABAQUS software and the total number of elements used for the model was 2600. For the critical analysis 600 elements were used to define the mesh in notch area as the notch area is the main concern of the investigation and this number of elements satisfy the computer programme constrains.

3.0 ELASTIC STRESS/STRAIN ANALYSIS OF THE SPECIMEN

3.1 Tensile Loading

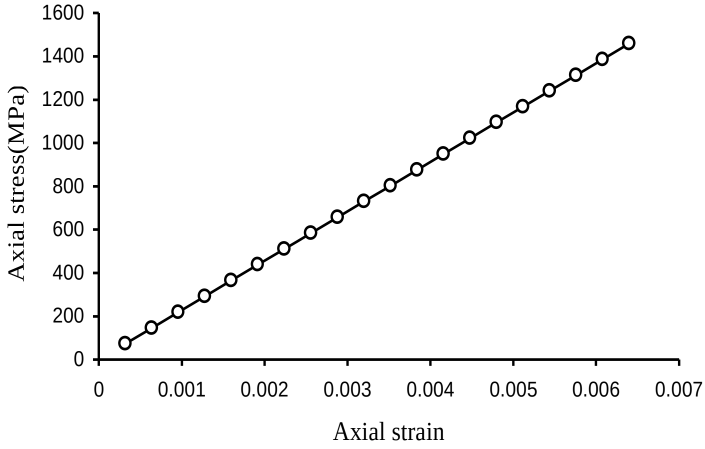

Fig. 3: Linear variation between axial stress / strain (S22-E22) at the notch root

A relationship between the stress and the strain at the notch root was found by applying a 0.2 mm axial displacement to the specimen. As shown in Fig. 3, it behaved linearly as expected in the elastic region. The gradient of the graph was calculated to be 208,000 MPa and it is close to the established values of modulus of elasticity.

In practice, fatigue failure usually occurs at notches or stress concentrations. Stress concentrations often result maximum local stresses, σ_{max}, at the discontinuities, which are many times greater than the nominal stress (S) of the member. In ideal elastic members, the ratio of these stresses is designated as K_t, the theoretical stress concentration factor Knop etal , 2000). However, there are a number of assumptions in the local strain approach that could contribute to the difference between low and high K_t specimens. One of these assumptions is that Neuber's rule can adequately estimate the stress and strain at the root of the notch (Peterson, 1974).

Fig. 4 shows the FEA results of (S22 direction) stress distribution throughout the specimen when a 0.2 mm axial displacement is applied under the elastic condition. Maximum stress can be observed at the notch root.

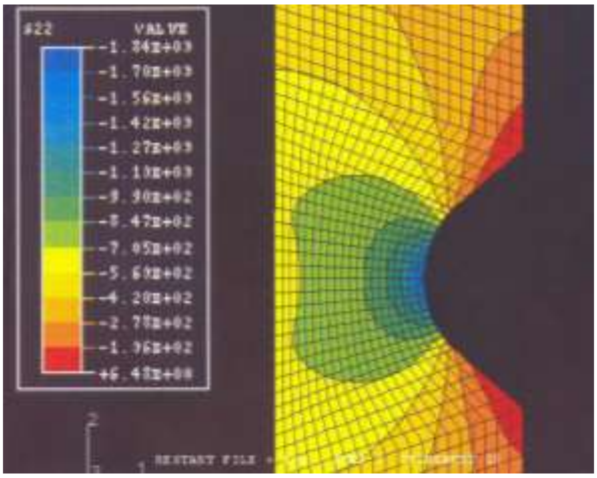

Fig. 4: The elastic stress distribution near the notch root

Several parts of the stress distribution in the vicinity of the notch root can be seen. The first zone includes the maximum stress. Behind the third zone, the stress values are comparatively small.

4.0 THEORETICAL STRESS CONCENTRATION FACTOR ANALYSIS:

An estimate to the magnitude of the stress concentration factor may often be obtained by undertaking a manual analysis. Considering the Figure 4, from the FEA elastic results, for an applied displacement of 0.2 mm, the theoretical stress concentration factor K_t was calculated and the value was 1.8, whereas the established K_t value from handbook (Knop et al , 2000) is 1.6 resulting 12.5% higher value.

5.0 ELASTIC/ PLASTIC STRESS ANALYSES:

Plasticity deals with the methods of calculating stresses and strains in an irreversibly deformed body after all elements of the body have yielded. It is necessary to establish equations of equilibrium and compatibility, and to determine the experimental relations between stress and increments of strain. The most difficult problem to solve in plasticity is those of constrained plastic flow (Buczynski, and Glinka, 1997). Total strain within elastic-plastic range can be derived as:

$$\varepsilon = \frac{\sigma}{E} + \left[\frac{\sigma}{K}\right]^{1/n}$$

Where 'E' is the modulus of elasticity, 'σ' is the direct stress, 'K' is strength coefficient and 'n' is strain-hardening exponent (Benham, et al 1996).

For the plastic analysis the model was subjected to isotropic hardening behavior and for the analysis yield stress of the material and the plastic strain limits were given as input to the software programme.

Plastic strain is the portion, which cannot be recovered on unloading. Plastic strain produces both changes in grain shape and, on a very much smaller scale, changes in the distribution of lattice defects or dislocations (Medagedara and Sarathchandra 1994). It is these changes that cause strain hardening. Plastic strain can produce changes in the yield point of single crystal of pure metal (Shang et al , 1999).

6.0 TENSILE LOADING

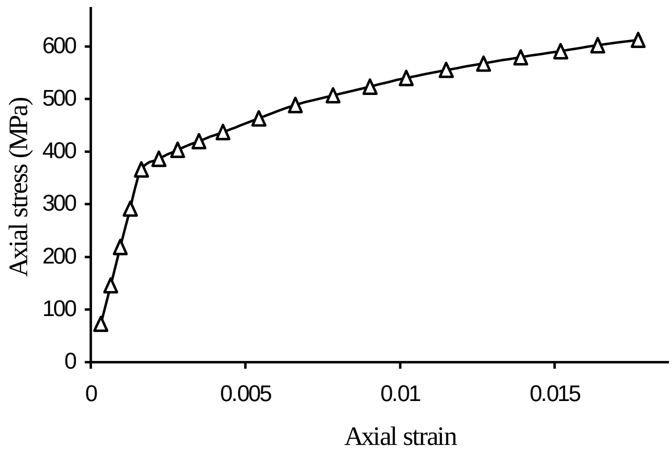

Fig. 5: The stress–strain response at the notch root for tension (0.2 mm displacement) in elastic-plastic condition

As in the Fig. 5, when applying an elastic-plastic load, the graph behaves linearly up to the stress of 350 MPa and then it changes to plastic behavior. Considering the linear portion, gradient of the graph was calculated to be 208,000 MPa, which is close to the modulus of elasticity of the material.

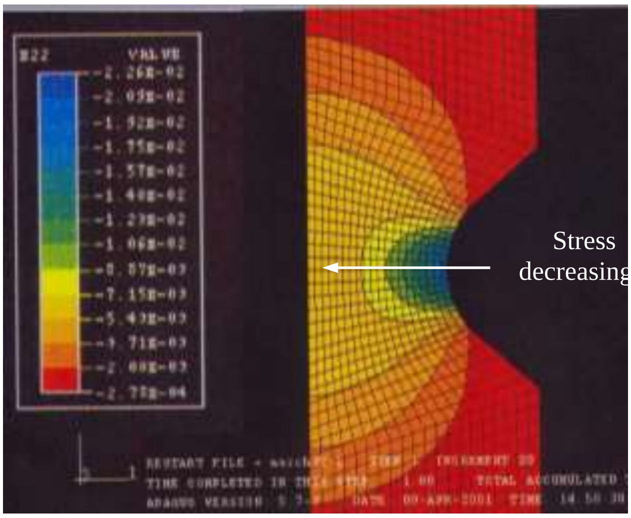

Fig. 6: Area of the plastic strain when applying 0.2 mm axial displacement

The elastic-plastic stress/strain distribution near the notch root exhibits first a peak point and then a decreasing dependence to the distance from the notch root. Few different stress/strain distributions in the vicinity of the notch root can be particularly distinguished as in Fig. 6.

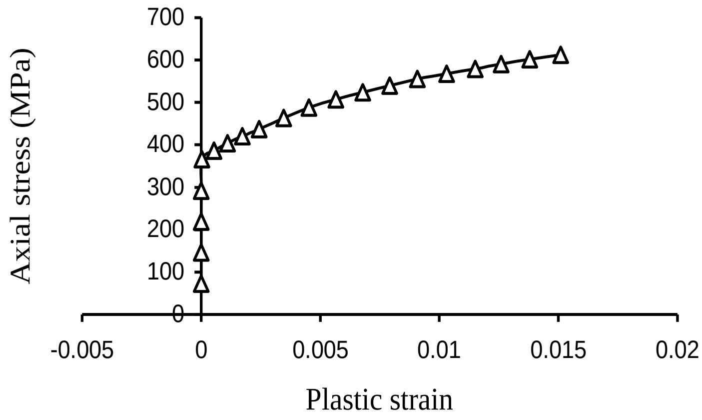

Fig. 7: Relationship between axial stresses against plastic strain (0.2mm axial displacement)

Fig. 7, shows the plastic strain, which cannot recover on unloading. Plastic strain starts from 350 MPa of axial stress and showing its continuous increase with the strain amplitude.

7.0 NEUBER ANALYSIS

7.1 Estimation of elastic-plastic stress concentration factor:

Elastic-Plastic stress concentration factor was estimated using FEA results and using Neuber equation $Kt^2Se = \sigma\epsilon$. Considering the Fig. 6, from the FEA results, for an applied displacement of 0.2 mm, the theoretical stress concentration factor K_t was calculated and the value was found as 1.6. This value is compatible with the theoretical K_t value given in stress concentration graph (Knop et al , 2000).

8.0 MODELLING CYCLIC STRESS-STRAIN HYSTERESIS BEHAVIOUR

Cyclic stress-strain curves are useful for assessing the durability of structures and components subjected to repeated loading. The response of a material subjected to cyclic inelastic loading is in the form of a hysteresis loop. The area within the loop is the energy per unit volume dissipated during a cycle. It presents a measure of the plastic deformation work done on the material.

8.1 Cyclic Axial load

For elastic plastic condition, a cyclic axial load was applied to the ABACUS model with a cyclic load (with reversible axial displacement) applied as axial displacement varying with time is shown in Fig. 8.

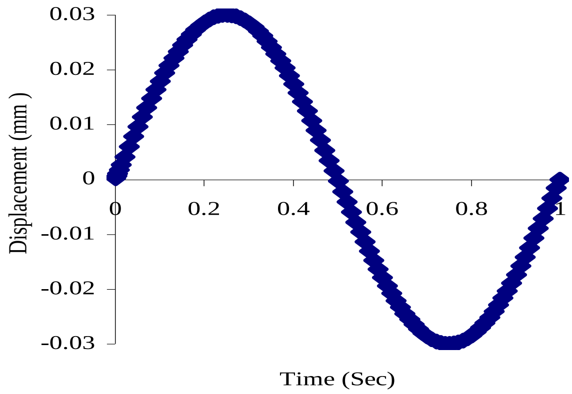

Fig. 8: Applied axial displacement against time for cyclic loading

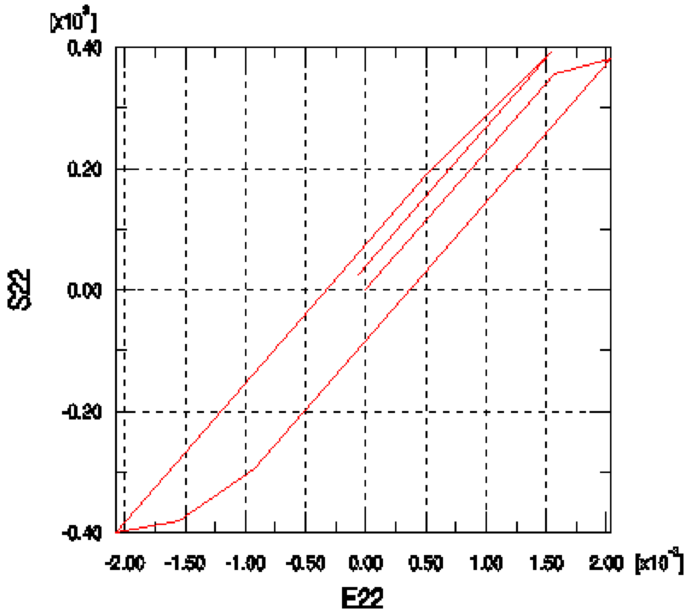

Fig. 9: Axial stress-strain (S22-E22) variation at the notch root (FEA results) for ±0.03 displacements (cyclic loading)

The Fig. 9 shows the hysteresis loop obtained from cyclic axial loading. The loop has a little plastic energy and the maximum strain range of the loop is 0.002. Using material cyclic stress-strain curve and Neuber curve, hysteresis loop was predicted for same cyclic axial load. Initially the hysteresis loop was developed for the material by using the equation, and developed the other curves by using Neuber equation $Kt^2Se = \sigma\epsilon$. For stress concentration factor K_t, a value of 1.6 was used and developed several curves by changing nominal stress value (S). Then common values were found from the both graphs to predict the Neuber hysteresis loop as shown in Fig. 10.

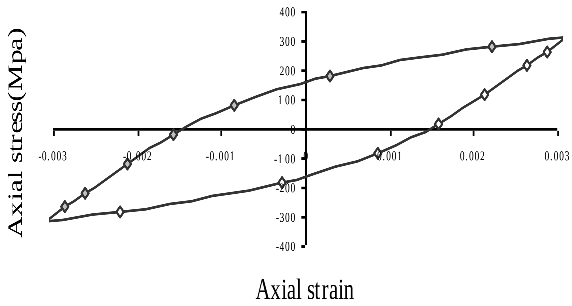

Fig. 10: Predicted hysteresis loop from Neuber equation for same FE load

Comparing the strain values obtained from the finite element analysis and the value achieved from the predicted hysteresis loop for cyclic loading condition, the strain predicted from Neuber was 36% than that obtained from finite element analysis.

9.0 CONCLUSIONS

An approximate model for elastic–plastic stress–strain analysis of a notched bar under variable amplitude tension and torsion cyclic loading is presented. A detailed FE analysis was performed for an axisymmetric steel bar of circular cross-section with a circumferential notch subjected to different elastic, elastic-plastic loading cases. The comparison of the finite element results with that of Neuber analysis shows that the latter results were over estimated than that of the FEA. ABAQUS software can provide sufficiently accurate estimations for elastic–plastic notch stresses and strains for elastic, elastic-plastic cyclic loading conditions. Neverthless, further research is required to consolidate the validity of results obtaned from finite element method.

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