

An Algorithm for the Holzer Method & Holzer Correction Formula for Torsional Vibration Analysis

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Abstract –. This paper investigates the development of an algorithm for the Holzer method and Holzer correction formula. Natural Frequency & Mode shapes are considered as dominant factors in a multi degrees of freedom system design. The Holzer method is used to calculate Natural Frequency mostly in torsional vibration systems, which is a tabular method for the analysis of multi mass lumped-parameter systems. It is considered as a trial and error method due to the time-consuming steps of calculation. When the number of rotors exceed than three it becomes extremely difficult to determine Natural Frequency in calculations by hand. In order to address these problems, an algorithm is developed to calculate Natural Frequencies. A user-friendly software interface is also integrated to enter parameters and variables of the torsional system. Using these data, the proposed algorithm is able to determine and displays whether the user given frequency is the natural one or not. If it is not the Natural Frequency, most accurate Natural Frequency can be obtained by the algorithm it-self. Further proposed software facilitates to calculate Natural Frequency in a given range in a user-friendly interface. By changing the parameters, it is able to obtain the relevant Natural Frequency which is necessary when optimizing a design. As well as the program produces the behavior of the system with change in parameters through graphs.

Keywords: Holzer Correction Formula, Holzer Method, Natural Frequency, Torsional Vibration System, Vibration

Nomenclature

i - number of rotors
 T - Torque (Nm)
 n - Final rotor
 G - Modules of rigidity (Nm⁻²)
 L - Length of the shaft (m)
 J - Polar moment of inertia of the shaft cross section (m⁴)
 d - diameters for different shaft lengths (m)

Greek Letters

ω - frequency (rad s⁻¹)
 θ - Angular position of the disk (rad)

Subscripts

I_i - moment of inertia of i^{th} rotor (kgm²)
 a_i - Angular Strains (rad)
 K_i - torsional stiffness of i^{th} shaft (Nmrad⁻¹)
 ω_n - Natural Frequency (rads⁻¹)

1 INTRODUCTION

Vibration affects the performance of systems favorably as well as unfavorably. Unfavorable vibrations are detrimental for systems, and designers often try to avoid them within their approaches for the design. The most harmful situation is that the acting frequency falls within the range of Natural Frequency. Therefore, the Natural Frequency of the system should be found first. The most common and traditional method is to build a physical model and follow the "trial and error" method. This is time consuming and the accuracy of those calculations depends on the performance of the person doing it. Second method is the modern approach to build an analytical model to represent the system and predict the consequences very early before the manufacturing process starts. In an analytical model, equations for each degree of freedom (DOF) are needed. This is not practical with systems which have more than three DOFs. To overcome this issue the Holzer Method is introduced to the design world. Instead of going through many calculations Holzer method allows us to determine Natural Frequency by filling a table of elements. Natural Frequency is the frequency at which a system tends to oscillate in the absence of any driving force. When a system vibrates at a frequency equivalent to its Natural Frequency, its vibration amplitude increases significantly which could lead to irreversible damage. Sometimes we may find different configurations of vibrations such as linear vibration, rotational vibrations and torsional vibration.

Torsional vibration is an oscillatory angular motion that can happen in mechanical systems when there is a non-uniformity of the torque developed or transmitted. It occurs in the shafting systems. Torsional vibration is gaining the attention of the industrial community because of its high influence in the machine components. Under some environments, rotational elements can fail due to torsional vibration. Torsional vibration is a complex vibration having different structures, with different parameters, the free vibration and forced vibration systems. This vibration cannot be detected easily. Even though its amplitude can be detected using special measuring equipment. Natural frequency is a main factor in a torsional system in order to control the torsional vibration. It is easy to calculate the Natural Frequency, if a torsional system consists of two or three rotors. When the number of rotors increase, it becomes extremely difficult to determine the Natural Frequency. Hence the Holzer method and Holzer correction formula is used. Natural Frequencies and mode shapes are determined by assuming a frequency and starting with a unit amplitude at one end. But, if the manual calculation is done, the major challenges of this method are high time consumption and even a small error affects the result.

In order to address these problems, in this study a software was developed. The user only needs to provide parameters of the torsional system. Using these data, the software program determines and displays whether the frequency which is given by the user is natural or not. If it is not a Natural Frequency, most accurate Natural Frequency can be obtained via the program. Further proposed algorithm facilitates to calculate Natural Frequency in a given range easily. Also, it provides all the frequencies and necessary values through a graph and tables for given range are provided. By changing the parameters, it is able to obtain the relevant Natural Frequency which is necessary to operate the torsional system. As well as the program produce a graphical behavior of the parameters. Hence the proposed algorithm can be identified as user-friendly software. J Quiroga *et. al.* (2019) has developed a system to check only the effectiveness of Holzer Method using ANSYS. According to the literature review this proposed algorithm is a novel method to find natural frequencies of complex torsional systems.

2 METHODOLOGY

The proposed algorithm is programmed in visual studio 2012 platform. Holzer method was used to develop the algorithm. Then mathematical calculations and relations were developed between parameters using visual basic codes.

A. Analytical models used for various situation

In the algorithm we considered two torsional vibration cases i.e. straight line free torsional vibration system and forced torsional vibration system of which typical schematic diagrams are shown in figure 1 and figure 2 respectively.

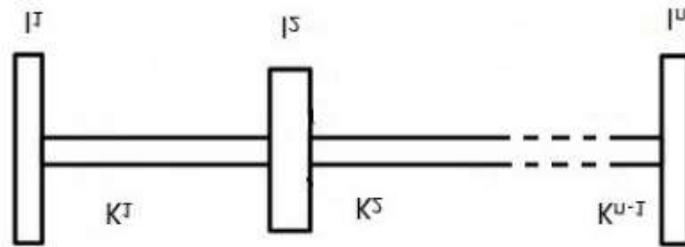


Fig. 1. Straight line free torsional vibration system

Table 1 Holzer table for free vibration

1	2	3	4	5	6	7	8
i	I_i	ω^2	a_i	$I_i \omega^2 a_i$	$[\sum I_i \omega^2 a_i] = Y_i$	K_i	$[(\sum I_i \omega^2 a_i / K_i)] = X_i$

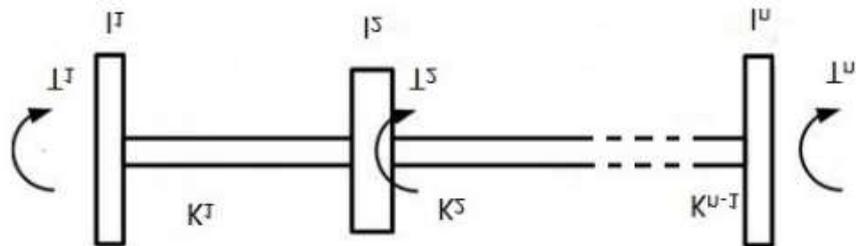


Fig. 2. Straight line forced torsional vibration system

Table 2 Holzer table for forced vibration

1	2	3	4	5	6	7	8	9
i	I_i	ω^2	a_i	$I_i \omega^2 a_i$	T	$[\sum (I_i \omega^2 a_i + T)] = Y_i$	K_i	$[\sum (I_i \omega^2 a_i + T) / K_i] = X_i$

The Holzer table prepared for two cases are shown in Table 1 and Table 2.

Angular Strains were calculated according to the Holzer method,

$$\begin{aligned} a_1 &= 1 \\ a_2 &= a_1 - X_1 \\ a_3 &= a_2 - X_2 \\ &\vdots \\ a_n &= a_{n-1} - X_{n-1} \end{aligned}$$

Via this software Natural Frequency can be obtained easily for straight line free and forced vibration torsional systems. If the frequency value which user entered is not the Natural Frequency, then a frequency which close to the Natural Frequency is provided to the user using Holzer Correction Formula. Then assume,

$$I_n = I^*$$

And I^* value was determined, assuming,

$$Y_n = 0, \text{ Then, } \delta I_n = I_n - I^*$$

using the correction formula,

$$\delta \omega^2 = \{[-\omega^2 (\delta I_n) a_n^2] / [r=1 \sum^n I_r a_r^2]\}$$

then corrected Natural Frequency ω_n , was obtained

$$\omega_n^2 = \omega^2 + \delta \omega^2$$

Figure 3 shows a typical schematic diagram for a straight line one end fixed torsional vibration system.

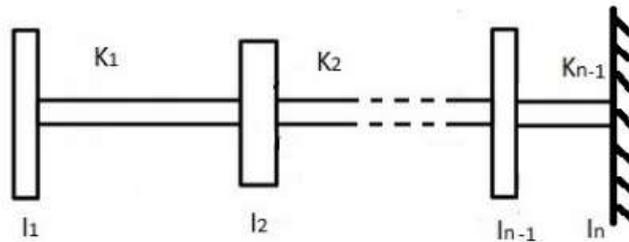


Fig. 3. Straight line one end fixed torsional vibration system

This system has only one end fixed and it is considered as a rotor having a very large moment of inertia. That is I_n has infinite inertia ($I_n \approx \infty$). In this system Natural Frequency can be obtained considering a_n . If the user entered a frequency value not equal to Natural Frequency, then a frequency which is closer to Natural Frequency is provided to the user using Holzer Correction Formula. Then assume,

$$K_{n-1} = K^*$$

And K^* value was determined, assuming, $a_n = 0$

Then,

$$\delta K_n = K_{n-1} - K^*$$

using the correction formula,

$$\delta \omega^2 = \{[(\delta K_n) a_n^2] / [\sum_{r=1}^{n-1} I_r a_r^2]\}$$

then corrected Natural Frequency ω_n , was obtained

$$\omega_n^2 = \omega^2 + \delta \omega^2$$

For example, to calculate the Natural Frequency for the given below rotor system as shown in Fig.4, Let us assume that the Natural Frequency of vibration is $\omega = 1.05 \text{ rad s}^{-1}$. That is $\omega^2 = 1.1 \text{ rad s}^{-1}$. $I_1 = 1 \text{ kgm}^2$, $I_2 = 2 \text{ kgm}^2$, $I_3 = 2 \text{ kgm}^2$, $a_1 = 1$ and $K_1 = 1$, $K_2 = 2$. If do the calculations manually, these steps need to be done.

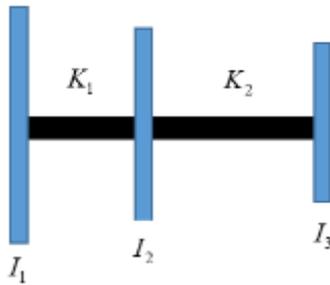


Fig. 4. Straight line free torsional vibration system

<p>1. $I_1 \omega^2 = 1.1$ $I_1 \omega^2 a_1 = 1.1$ $\Sigma I_1 \omega^2 a_1 = 1.1$ $\frac{1}{K} \Sigma I_1 \omega^2 a_1 = 1.1$</p>	<p>2. $I_2 \omega^2 = 2.2$ $a_2 = a_1 - [\frac{1}{K} \Sigma I_1 \omega^2 a_1]$ $= -0.1$ $I_2 \omega^2 a_2 = -0.22$ $\Sigma I_2 \omega^2 a_2 = 0.88$ $\frac{1}{K} \Sigma I_2 \omega^2 a_2 = 0.44$</p>	<p>3. $I_3 \omega^2 = 2.2$ $a_3 = a_2 - [\frac{1}{K} \Sigma I_2 \omega^2 a_2]$ $= -0.54$ $I_3 \omega^2 a_3 = -0.19$ $\Sigma I_3 \omega^2 a_3 = -0.31$</p>
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Table 3 Holzer table for given example

i	I_i (kgm ²)	$I_i \omega^2$	a_i	$I_i \omega^2 a_i$	$\Sigma I_i \omega^2 a_i$	K_i	$\frac{1}{K} \Sigma I_i \omega^2 a_i$
1	1	1.1	1.0	1.1	1.1	1	1.1
2	2	2.2	-0.1	-0.22	0.88	2	0.44
3	2	2.2	-0.54	-0.19	-0.31		

As shown in Table 3, the sum of values of the column $\Sigma I_i \omega^2 a_i$ is not equal to 0 hence the value of $\omega = 1.05 \text{ rad s}^{-1}$ cannot be the correct Natural Frequency. Therefore, an additional calculation has to be done in order to find the correct Natural Frequency. There are two methods this can be implemented. In analytical method the same calculation process needs

to be done at least twice to get a sufficiently close value to the correct Natural Frequency assuming another frequency value as a Natural Frequency. In the other method which is the Holzer correction formula also requires additional calculation. If do the calculations manually for Holzer correction formula, these steps need to be done. Since the residual torque is not zero, $\omega = 1.05 \text{ rad s}^{-1}$ cannot be the correct frequency

Let us say $I_3 = I^*$ and determine the value of I^* so that $\sum_{i=1}^3 I_i \omega^2 a_i = 0$

at the same frequency ($\omega = 1.05 \text{ rad s}^{-1}$) and the same ar. then modified row 3 gives,

3	I^*	$1.1 I^*$	-0.54	$0.54 I^*$	$0.88 - 0.594 I^*$		
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Therefore,

$$\begin{aligned} 0.88 - 0.594 I^* &= 0 \\ I^* &= 1.48 \end{aligned}$$

This means that when $I^* = 1.48 \text{ Kgm}^2$, the exact Natural Frequency $\omega = 1.05 \text{ rad s}^{-1}$, Now the we can use the correction formula to determine the Natural Frequency of given system

$$\delta I_3 = I_3 - I^* = 2 - 1.48 = 0.52 \text{ Kgm}^2$$

Using the correction formula

$$\begin{aligned} \delta \omega^2 &= \frac{-\omega^2 (\delta I_n) a_n^2}{\sum_{r=1}^3 I_r a_n^2} \\ &= \frac{1.1 \times 0.52 \times (0.54)^2}{[1 \times 1 + 2 \times (0.1)^2 + 2 \times (0.54)^2]} = -0.104 \end{aligned}$$

Corrected frequency ω_n is given by,

$$\begin{aligned} \omega_n^2 &= \omega^2 + \delta \omega^2 = 1.1 - 0.104 \\ \omega_n &= \sqrt{0.996} = \underline{\underline{0.998 \text{ rad s}^{-1}}} \end{aligned}$$

Note - In case the value for I^* was obtained as negative, the same process has to be executed for a fresh value for ω_n again.

If the manual calculation was done as above, the major challengers of this method are high time consumption and even a small error affects the result. In order to address these problems, proposed software can be used.

B. Natural Frequency obtain using graphical method

If the user entered two frequency square values (ω_1^2 and ω_2^2) and their residual torque ($[\sum I_i \omega^2 a_i] = Y_i$) Y_1 and Y_2 , then Natural Frequency can be obtained using the graphical method as shown in Fig.5

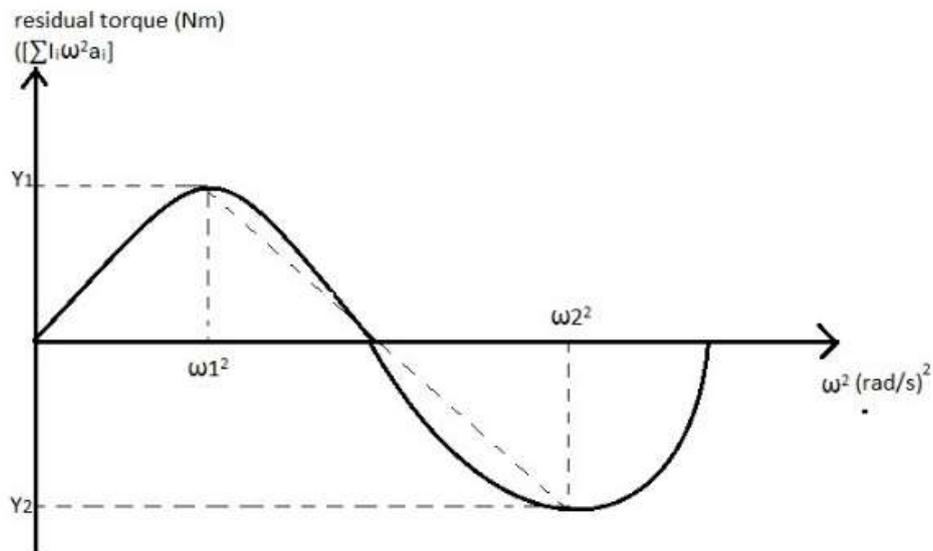


Fig. 5. Natural Frequency using graphical method

C. Calculation of Natural Frequency by changing different parameters

Calculation of Natural Frequency by changing different parameters like I_i and d_{i-1} in a torsional system is the best method to keep the acting frequency away from it and make it easy to select frequency of the motor too. From this proposed software facilitates to change parameters I_i and d_{i-1} for given range and obtain residual torque $(\sum I_i \omega^2 a_i)$ values in a graph. Through these observations a proposed system can be optimized according to design requirements.

3 SIMULATION EXPERIMENTS AND RESULTS

A. The developed algorithm consists four main options as follows, Natural Frequencies of straight line free torsional vibration system, Natural Frequencies of straight line free torsional vibration with one end fixed system, straight line forced vibration system and Natural Frequencies using graphical method. Parameters need to provide to find the Natural Frequency. Then the proposed algorithm determines and displays Holzer table with the values. Fig.6 shows the way Holzer table values are displayed in the graphical user interface. If it is not the Natural Frequency, most accurate Natural Frequency can be obtained by the program quickly and easily.

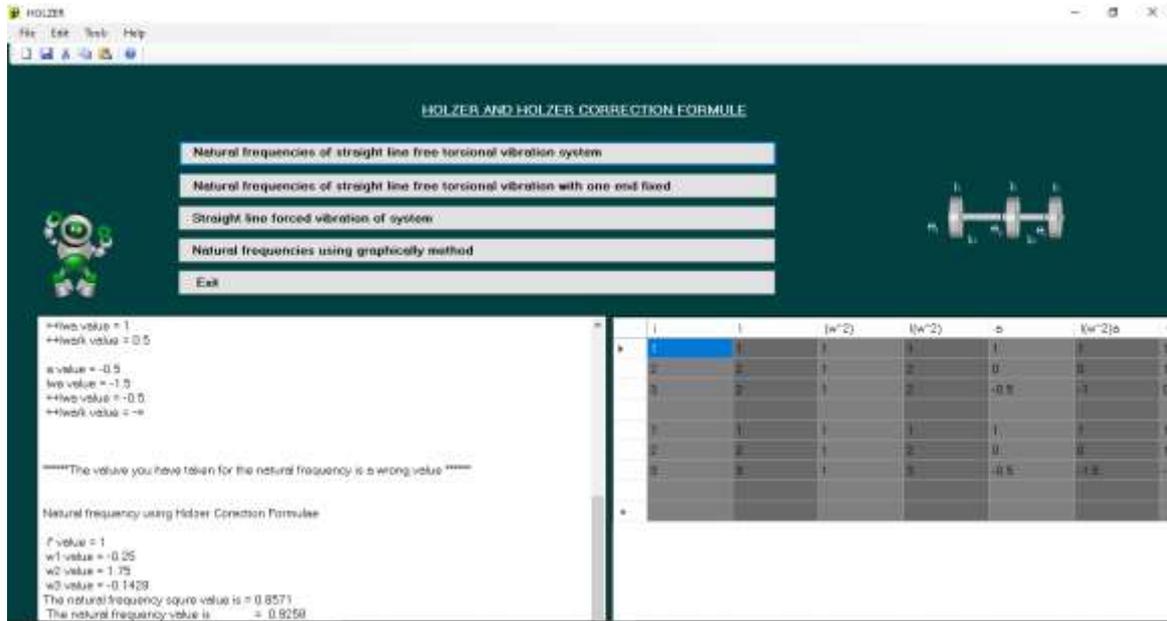


Fig. 6. Holzer table with the values

B. Also proposed software facilitates to calculate Natural Frequency in a desired range. By changing the parameters in the individual components of the system, the necessary graphs and details can be obtained. Because of this option we don't need to make more Holzer tables for each frequency to seek the natural frequency. We can obtain all the necessary accurate details from the graph and tables for a given range. The proposed program determines the Holzer graph and table for each frequency in the given range as shown in Fig.7 and Fig.8.

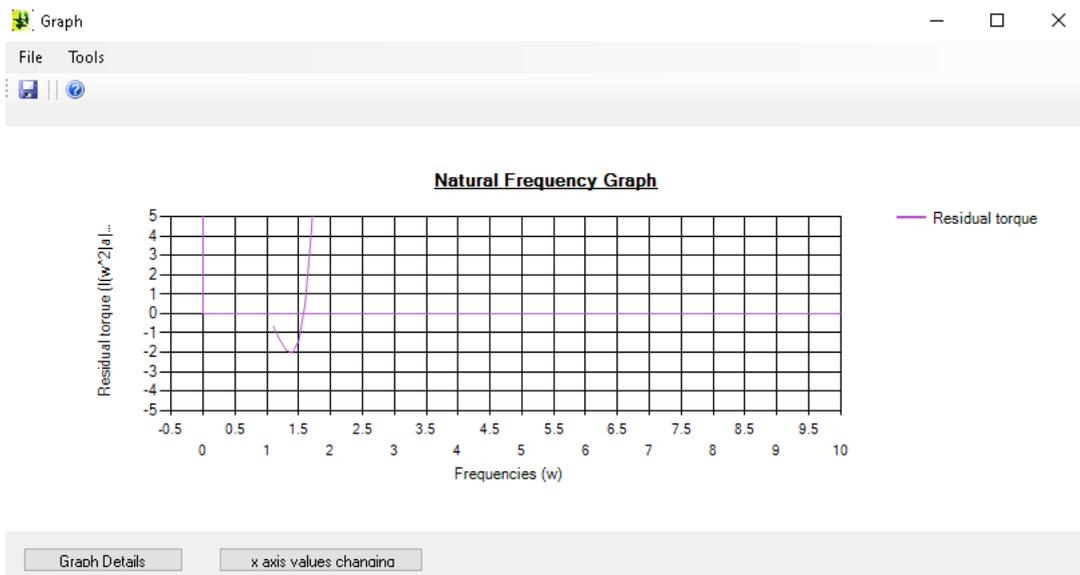


Fig. 7. Displaying all the details in a Graph for given range

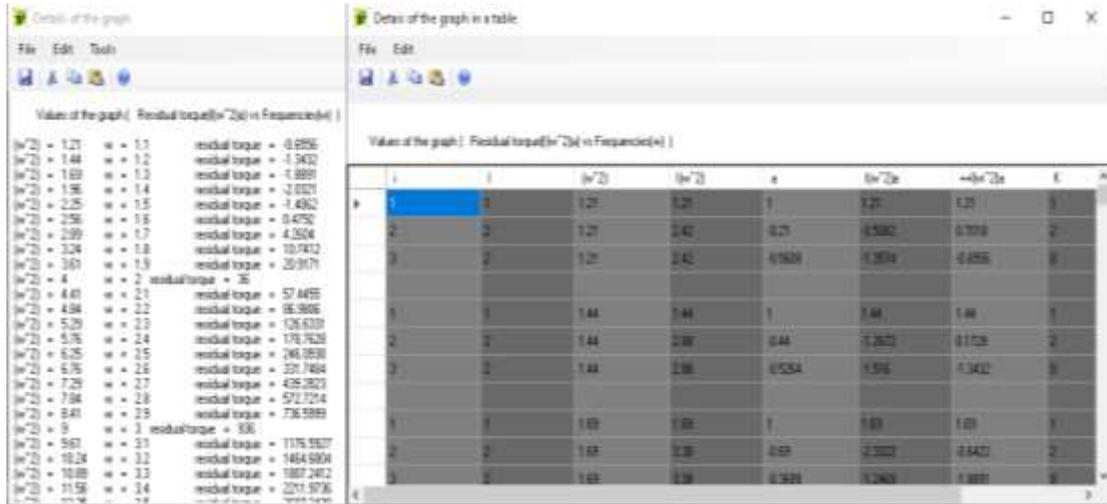


Fig. 8. Displaying all the details and Holzer tables for given range

Further proposed algorithm facilitates to change the selected range of the graph to observe data closely as shown in Fig.9.

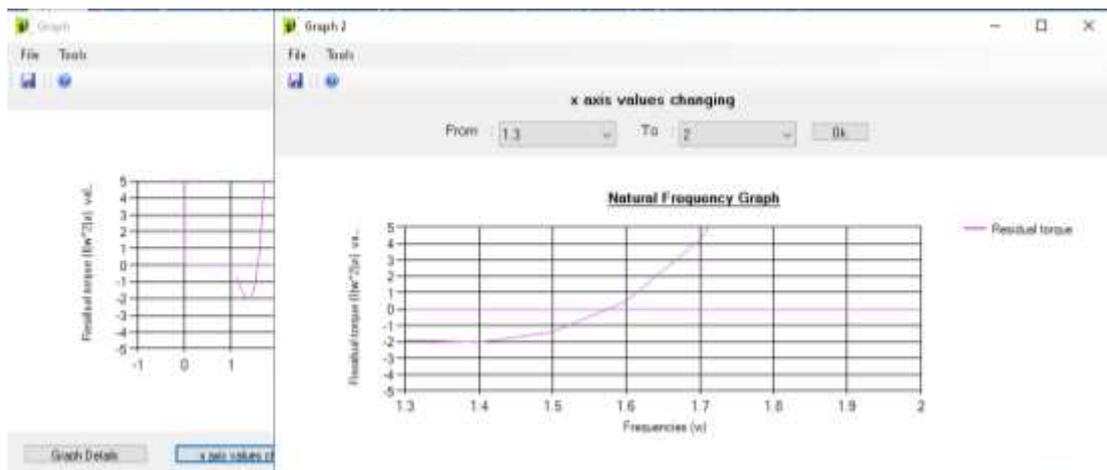


Fig. 9. Changing the range of the frequency

C. The graphs can be plotted with different parameters. From this proposed software we can obtain necessary parameter values by changing parameters for the given range. As shown in Fig.10 Holzer tables and graphs can be obtained according to the range of frequency(ω^2).

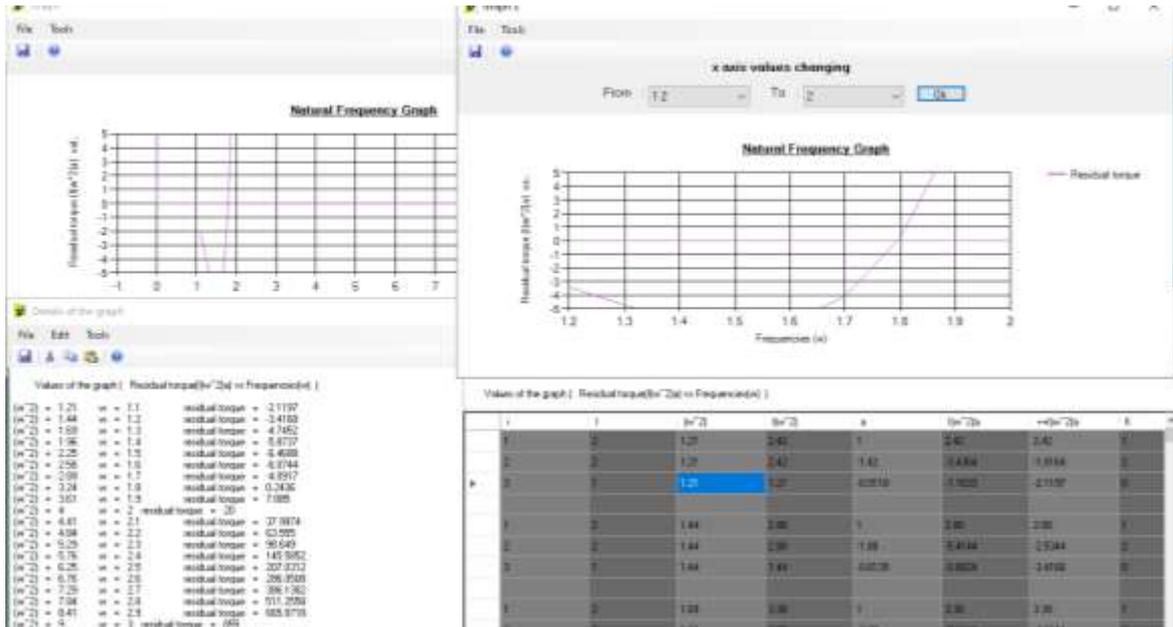


Fig. 10. The details, Holzer tables and graphs according to the range of frequency (ω^2)

As shown in Fig.11 Holzer tables and graphs can be obtained according to the range of moment of inertia (I).

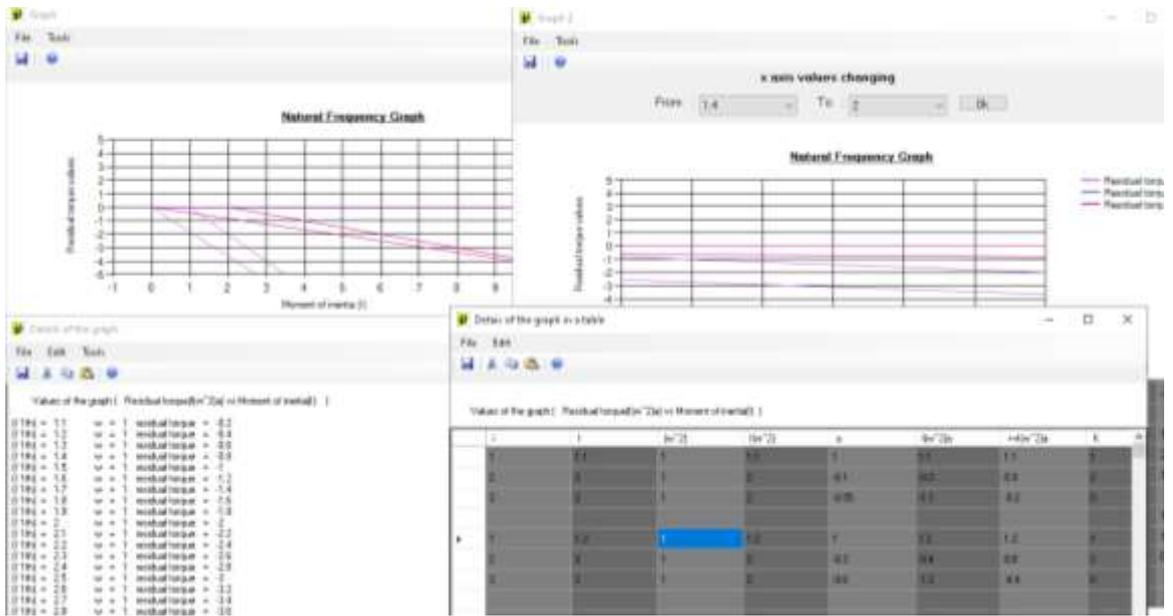


Fig. 11. the details, Holzer tables and graphs according to the range of moment of inertia (I)

As shown in Fig.12 Holzer tables and graphs can be obtained according to the range of diameter and length of shaft.

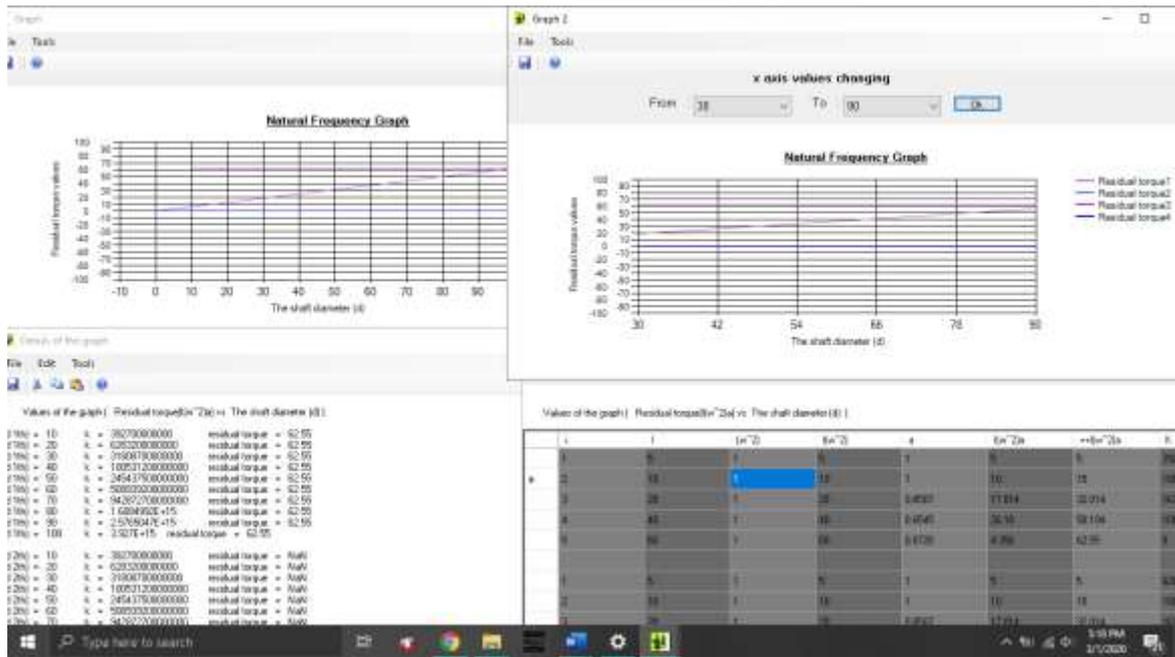


Fig. 12. The details, Holzer tables and graphs according to the range of diameter and length of shaft

D. This proposed software facilitates to save all the tables, graphs, and other details in any format.

4 CONCLUSIONS

In this study we developed an algorithm to facilitate torsional vibration with multi degree of freedom. In industry there are different types of torsional vibration systems and for the performance and stability of such systems, vibration is directly affected. When vibration amplitude increases significantly, that could lead to catastrophic failures. Therefore, the system should be modified to drive the shaft with a suitable motor frequency according to the application, which leads to minimum risk of failure. The important parameter is the Natural Frequency of the system and it should be calculated with a high accuracy. Performing manual calculation is a tedious process where repeating of calculation involves and hence time consuming. When the number of rotors exceed than three it becomes extremely difficult to determine Natural Frequency by manual calculations. This proposed algorithm facilitates to find Natural Frequencies in straight line free or forced torsional vibration systems and one end fixed systems. The proposed algorithm is capable of calculating Natural Frequency faster and accurate.

In future it is expected to develop the algorithm to accommodate systems with torsional vibration of a branched system, forced torsional vibration of a branched system, torsional vibration of a branched one end fixed system, forced torsional vibration of a branched one end fixed system and damped torsional systems.

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