RESEARCH ARTICLE *OUSL Journal, 2023*
https://doi.org/10.4038/ouslj.v18i1.7569 *Vol. 18, No. 1, (pp. 33-60)*

An Optimization Model for the Hard Time Windows Vehicle Routing Problem with Moving Shipments at the Cross Dock Center

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Abstract

Cross-Docking (CD) technique was initiated in the 1930s to make a cost-effective supply chain. Vehicle Routing Problem (VRP) is one of the widely discussed optimization problems. The research on integration of VRP with CD (VRPCD) was initiated at the beginning of 2000s. Moving Shipments (MS) from receiving doors to shipping doors is an activity inside a Cross-Dock Centre (CDC). This study mainly considers MS as an additional aspect in the literature of VRPCD. In this study, not only loading or unloading shipments at all the nodes including CDC and homogenous fleets of vehicles within pickup or delivery process are considered, but also aspects of heterogeneous fleets of

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(Received 17th September 2022; Revised 23rd June 2023; Accepted 26th June 2023) © OUSL)

vehicles between pickup and delivery processes are considered. Furthermore, Time Windows (TW) characteristics are also considered here. A mixed integer nonlinear programming model is developed to obtain the optimal solutions to hard time windows vehicle routing problem with moving shipments at the cross-dock centre (TW-VRPCD-MS). The compatibility of the proposed model is tested using sixteen randomly generated small scale instances. Since the average computational time is reasonably less for the tested instances, it can be concluded that this proposed model can be used for last time planning for similar small-scale problems. Further analysis revealed that the convergence rate to reach the optimal solution rises exponentially with the scale of the problem. Therefore, this study recommends in applying heuristic or metaheuristic techniques to solve large scale instances of TW-VRPCD-MS to obtain a near optimal solution in a reasonable computational time.

Keywords: *cross- docking, moving shipments, time windows, vehicle routing*

Introduction

Sustaining a commercial enterprise in the highly competitive global market of today requires products be supplied and delivered on time to the right location at a lower cost. Hence, an efficient Supply Chain (SC) plays an important role in this regard. In a typical SC, suppliers/ manufacturers, distribution centers/ warehouses/ transshipment centers and customers/ retailers/ vendors are the main elements. The integration and coordination of the activities among these elements are increasingly important to significantly reduce the cost involved in it. In traditional warehousing, receiving, sorting, storing, order picking and shipping are the key activities. More than 30% of the cost of the products is incurred due to these activities at the distribution centers (Apte & Viswanathan, 2000).

To reduce this extra cost, a modern warehousing technique that could optimize SC is needed. Therefore, creative docking strategy known as Cross- Docking (CD) technique was introduced in the 1930s. However, it only became popular from the 1980s after the successful application at Walmart (Apte & Viswanathan, 2000). By employing this CD technique, market shares and the profitability of Walmart Company improved (Stalk et al., 1992). As a result of the utilization of this technique, other organizations such as *Toyota*, *Goodyear GB Ltd*, *Eastman Kodak Co*, *LLC* and *Dots* also reached their anticipated targets (Van Belle et al., 2012). In this study, CD technique employs distribution centers called as Cross Dock Centers (CDC). In these CDC, products received through indoors of CDC are loaded immediately within 24 hours through outdoors of CDC. Since, receiving sorting and shipping are the key functions at a CDC and storing and order picking causes more cost at a traditional distribution center, CDC can reduce the distribution cost significantly. Therefore, up to 70% of the cost of warehousing could be reduced by implementing CD technique in SC (Vahdani & Zandieh, 2010). In reality, not every product needs to be distributed through a CDC, but fast-moving products with constant demand, perishable products that need immediate shipment are more suitable to distribute through CDC. Therefore, this CD technique is mostly applicable to products such as pharmaceutical goods, vegetables and flowers, frozen food and dairy products and it would also be crucial for distributing companies of beverages, courier service providers and e-commerce organizations. Most of the organizations expect services (collection or distribution) in a specific time intervals to continue their day-to-day work with minimum disturbance. Therefore, serving those customers in that stipulated time frame with the optimized cost is another challenging problem in the SC.

Out of the three levels of decision at CDC, this study concerns only on the operational level, and it has many issues to be solved. Among them, routing between suppliers, CDC and customers is the primary issue considered in this study. In the field of Operations Research, Vehicle Routing Problem (VRP) is one of the mostly studied combinatorial optimization problems and it plays a significant role to make SC optimum and efficient. In 1959, Dantzig and Ramser proposed the VRP (Dantzig & Ramser, 1959). Among the several variants of VRP, this study focuses on VRP with Time Windows (VRPTW) which is an extension of capacitated VRP. VRPTW is NP-hard in the strong sense (Toth & Vigo, 2002). Nevertheless, VRPTW is a more realistic problem, and it has been studied widely. The soft TW can be violated with a penalty cost, but hard TW do not allow serving supplier/ customer after the upper bound of the window. However, in hard TW, if any vehicle arrives before the lower bound of the window, it must wait until it reaches the lower bound.

In 2006, Lee et al. (2006) initiated the research on VRP with CD (VRPCD). Since then, many researchers and practitioners have paid a lot of attention to VRPCD. The past studies show that research on transportation problem combined with CD has increased in frequency. From the literature reviewed on CD by Mavi et al. (2020), it was concluded that more than 85% of those research papers were published after 2004. Buakum & Wisittipanich (2019) recommended through a literature survey that, there was a need to focus on internal operations between indoors and outdoors of a Cross Dock Center when conducting research on Vehicle Routing Problem with Cross Docking. In Cross Dock Center, many internal activities take place. Unloading products from inbound vehicles, moving unloaded products from indoors of Cross Dock Center to outdoors of Cross Dock Center and reloading products to outbound vehicles are some of them. Therefore, this study focuses on Vehicle Routing Problem with Hard Time Windows by considering Moving Shipments inside a Cross Dock Center which includes the above-mentioned activities.

The contribution of this study is four-fold. Firstly, in this study, moving shipment from indoors after unloading the products from inbound vehicle to outdoors, to upload them to outbound vehicles is taken into consideration. Though the concept of moving shipments inside a cross-dock was implemented in 2022 by Gnanapragasam & Daundasekera (2022) to a capacitated VRPCD, it does not take the TW property into account. Also, almost all the past studies on Vehicle Routing Problem with Cross-Docking and Time Windows do not focus on internal operations between indoors and outdoors of a CD center.

Therefore, incorporating the Moving Shipments concept with the Time Window property is a novel idea in the literature of VRPCD. Secondly, time and cost for loading and unloading at doors of CD center in addition to all other nodes are also considered. Thirdly, two different sets of homogenous fleets of vehicles for pickup and delivery process but heterogeneous fleets of vehicles between pickup and delivery processes aspect are considered in the Time Windows variant of Vehicle Routing Problem with Cross-Docking. Finally, simultaneous starting time to load the products to outbound vehicles is also taken into consideration. The integrated four ideas is the new thought in the field of Vehicle Routing Problem with Cross-Docking and Time Windows.

The rest of this paper is organized as follows: the past studies are reviewed in the section 2. Section 3 explains the problem considered in this study. The formulation of the model with notations and the method of solution to the model are described in section 4. In section 5, the results of randomly generated small-scale instances with an example to illustrate the relevant components of time and cost are discussed. Conclusions based on the outcome of the study and recommendations for future research are proposed in section 6.

Review of literature

The first research on integration of VRP with CD was initiated in 2006 by Lee et al. (2006). The objective of the study was to determine the optimal vehicle routing schedule and number of vehicles in VRPCD while minimizing the cost of transportation and the fixed cost of vehicles used in the process. Two different sets of homogeneous fleets of vehicles were used in both pickup and delivery processes. The simultaneous arrivals of inbound vehicles to CDC are assumed. In 2010, Liao et al. (2010), with similar model from the study Lee et al. (2006), applied a modified algorithm and obtained relatively better solution with lesser computational time than that of Lee et al. (2006). Later in 2012, Vahdani et al.(2012) developed a hybrid method, and yielded significantly better solution than the algorithm presented by Lee et al. (2006). Another algorithm was employed by Yin & Chuang

(2016) and this algorithm outperformed the results of Liao et al. (2010). Through a meta-heuristic method proposed in 2014 by Yu et al. (2014), obtained a better solution for most of the instances in Liao et al. (2010).

Recently, Gunawan et al. (2020a) proposed another meta-heuristic algorithm which decomposed into two phases and this method outperformed the state-of-the-art algorithms in terms of solution and computational time. Improved best known solutions were obtained with benchmark instances by Gunawan et al. (2020b). In the following studies, additional characteristics were taken into account with the similar assumptions made by Lee et al. (2006): Hasani-Goodarzi & Tavakkoli-Moghaddam (2012) considered multi-product, heterogeneous inbound vehicles and splitting pickup and delivery properties. Heterogeneous property was considered in the studies Yin & Chuang (2016) and Birim (2016). The open configuration network was introduced by Yu et al. (2016). Multi product property was added by Gunawan et al. (2020).

Wen et al. (2009) proposed a generalized VRPCD in 2009 as an extension of the primary study of Lee et al. (2006). On the one hand, the condition that govern simultaneous arrivals of inbound vehicles were relaxed and on the other, the dependency rules and consolidation decisions such as corresponding relationships between suppliers and customers, the same fleet of vehicles used for both pickup and delivery processes were included in it. Moreover, the Time Windows in VRPCD were introduced by Wen et al. (2009). In a multi-source VRPCD with similar assumptions of Wen et al. (2009) by Tarantilis (2013), not only open and closed network configurations but also same and different sets of vehicles in both pickup and delivery processes were compared. In 2014, Morais et al. (2014) proposed a heuristic algorithm with six local search procedures to the model developed by Wen et al. (2009). This constructive algorithm is different from previously proposed algorithms and better solutions could be found for most of the instances given by Wen et al. (2009) and Tarantilis (2013).

Nikolopoulou et al. (2016) introduced further generalized VRPCD by extending one-to-one correspondence between suppliers and customers, which was similar to the study by Wen et al. (2009), to many-to-many correspondence between them. In addition, two distinct fleets of vehicles in both processes were also considered in its model. Its adaptive memory programming for one-to-one correspondence provided similar solutions to the study of Wen et al. (2009) and more promising solutions than the solutions obtained in the study of Morais et al. (2014). Exact solutions to small-scale instances of Wen et al. (2009) were tried out by Santos et al. (2011a) using Branch and Price (BP) algorithm. To avoid the symmetrical solutions in the study Santos et al. (2011a), a novel exact algorithm was presented by Santos et al. (2011b) by introducing a newer column generation (CG) technique. Grangier et al. (2017) introduced a matheuristic-based method and this method enhanced many of the previously best-known results.

In the following studies, modified models were developed by incorporating additional characteristics to the model of Wen et al. (2009). Fakhrzad & Sadri Esfahani (2014) added the simultaneous arrivals of vehicles and soft time windows. Split delivery was allowed in the study of Moghadam et al. (2014). (Larioui et al., 2020) extended the one-to-one correspondence between suppliers and customers by allowing customers to order from more than one supplier and compared the results within a few solution methods. A hybrid method was applied to a variant of VRPCD that maximizes the total profit of the CD system in the study of Baniamerian et al. (2018a). Baniamerian et al. (2018b) considered the customer satisfaction factor for VRPCD with time windows.

Statement of the problem

This study is the extension of Vehicle Routing Problem with Cross-Docking and considers mainly the Time Windows and Moving Shipments inside Cross Dock Center aspects into account. This integrated model is referred to it as Hard Time Windows Vehicle Routing Problem with Moving Shipments at the

Cross Dock Center (TW-VRPCD-MS). Figure 1 demonstrates the basic structure of the pickup and delivery processes coupled with CDC:

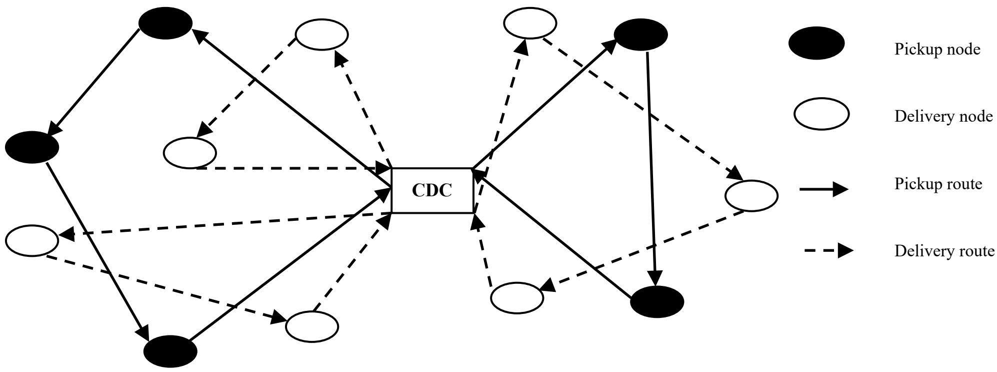


Figure 1. Basic structure of VRPCD

Assumptions of model

- Single CDC is considered, and each vehicle is assumed to be allocated to this specific CDC
- All suppliers produce only a single product
- Direct shipments from suppliers to customers are not allowed
- All vehicles should start and finish their routes at CDC
- The capacity of CDC is always adequate
- Homogenous fleets of vehicles within pickup or delivery process but heterogeneous between pickup and delivery processes
- Split pickup or delivery is not allowed, and each node must be served by exactly one vehicle within its time window
- The delivery process should be started after arriving all the products to the outdoor of CDC
- The accumulated quantity of each route must not exceed the total capacity of the vehicle

- The time horizon for whole transportation operation must be acknowledged

Integrated processes in TW-VRPCD-MS

Figure 2 represents the entire process of the Hard Time Windows Vehicle Routing Problem with Moving Shipments at the Cross Dock Center problem and followed by the procedure to explain all three processes in the integrated model in detail.

In this closed TW-VRPCD-MS problem, all the vehicles start their routes from CDC. The **pickup process** is taken place as follows. The randomly selected node is assigned to an inbound vehicle which respects its time window. Once the node is reached, *arrival time* to the node is determined only by the travelling time. Further, preparation time and time per pallet to load the products have to be considered in order to calculate the *service time* spent at each supplier node. The *departure time* from the node is determined by adding arrival time to the node and the service time at the node. Now from already selected node, another randomly selected node is assigned to the same inbound vehicle provided that already chosen inbound vehicle does not exceed its capacity and it satisfies the time window. If the capacity of the inbound vehicle exceeds or it violates the time window, the new node has to be assigned to another new inbound vehicle which also should start its route from CDC. These procedures are continued until all the suppliers are assigned to any of the inbound vehicles used at the pickup process.

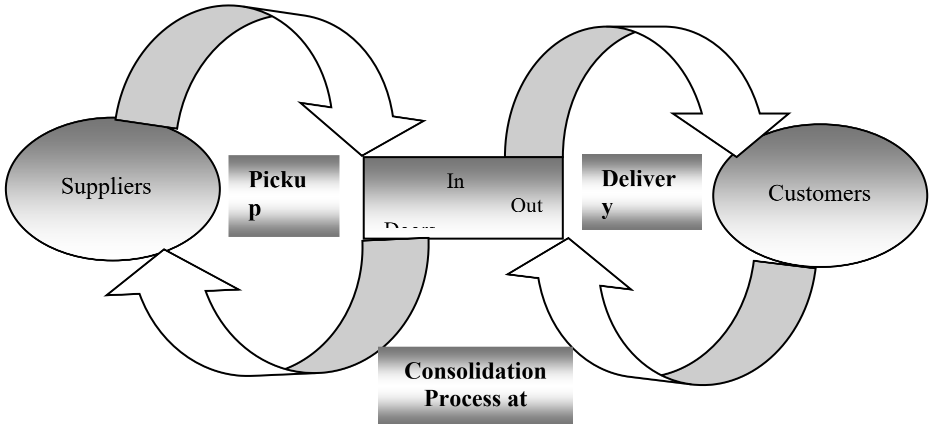


Figure 2. Integrated processes of TW-VRPCD-MS

In the **consolidation process**, *arrival time* of each inbound vehicle from the pickup process is calculated when it reaches the indoors of CDC after collecting products in its particular route. Subsequently, after reaching the indoors, preparation time and time per pallet to unload the collected products from pickup process in a particular route are applicable to determine the *unloading time* at indoors of CDC. Consequently, unloaded products at CDC are moved from indoors to temporary storage area located at CDC closer to outdoors. Here, time per pallet to move the shipments is applicable to calculate the *moving time* from indoors to outdoors at CDC and to calculate the *ready time* to start the loading at outdoors in CDC. It is assumed that the outbound vehicle cannot start reloading until all the products are moved to the temporary storage area from the indoors. Therefore, the ready time at the outdoors is the time at which orders are ready to be reloaded in their corresponding outbound vehicles. Next the demands of customers are consolidated according to their requests. Also in this case, *loading time* at outdoors of CDC which is like that as *unloading time* at indoors is determined.

Similar procedure in the pickup process is followed in the **delivery process** as well with a different set of homogeneous fleets of outbound vehicles, but with different capacity from the set of homogeneous fleets of inbound vehicles. In pickup and delivery processes, all the nodes (suppliers or customers) have their own *time windows* so that the vehicles must be arrived at those nodes only in that particular time intervals. Finally, after completing the delivery at the last node in that route, the outbound vehicle should return to CDC. It is emphasized that the entire process should be completed within the whole *time horizon* and it is assumed here as 16 hours.

Factors of total cost in TW-VRPCD-MS

The factors considered for the determination of total cost, which is to be minimized, are categorized as follows. *Transportation cost*: cost of travel in between nodes including CDC. *Service cost at nodes*: the cost for loading or unloading products at the pickup or delivery nodes respectively. *Service cost at CDC*: the cost for unloading accumulated products (collected through a particular pickup route) from an inbound vehicle at the indoors of CDC and cost for loading products (to be distributed in a particular delivery route) to an outbound vehicle at the outdoors of CDC. *Moving Shipments cost*: the cost of moving the unloaded products from each inbound vehicle at the indoors of CDC to load them to each outbound vehicle at the outdoors of CDC. *Vehicle operational cost*: the cost for maintaining or hiring the vehicles. It is to be noted that, service cost at CDC and cost of moving shipments inside CDC are considered as the cost due to the activities inside CDC.

Notations and Formulation

Notations

Indices

i, j	:	Indices for pickup or delivery nodes
h	:	Index for indoors or outdoors of CDC
k	:	Index for vehicles

Sets

$P = \{P_1, P_2, ..., P_n\}$	:	Set of n pickup nodes
$D = \{D_1, D_2, ..., D_{n'}\}$	:	Set of n' delivery nodes

$N = P \cup D$: Set of $(n+n')$ pickup and delivery nodes

$V_p = \{v_1^p, v_2^p, \ldots, v_m^p\}$: Set of m inbound vehicles

$V_d = \{v_1^d, v_2^d, \ldots, v_{m'}^d\}$: Set of m' outbound vehicles

$V = V_p \cup V_d$: Set of $(m+m')$ inbound and outbound vehicles

$O = o \cup o'$: Set of receiving (o) and shipping (o') doors of CDC

Parameters

tt_{ij} & tc_{ij} : Travelling time and travelling cost from node i to node j respectively

q_i : Quantity (supply/ demand) at node i

Q_p & Q_d : Maximum capacity of inbound and outbound vehicles respectively

m & m' : Number of used inbound and outbound vehicles respectively

oc^k : Operational cost of vehicle k

ST_i^k & SC_i^k : Service time and service cost at node i by vehicle k

DT_i^k & AT_i^k : Departure and arrival time of vehicle k at node i

$RT_{o'}$: Ready time at outdoors of CDC to load the quantity to outbound vehicles

A_t & A_c	:	Fixed time and cost of preparation for loading/ unloading products at nodes
B_t & B_c	:	Variable time and cost of loading/ unloading a pallet of the product at nodes respectively
T	:	Total time of the entire process
a_i & b_i	:	Lower and upper bounds of the time window of node i respectively

Variables

$$x_{ij}^k = \begin{cases} 1 \ , & \text{if vehicle } k \text{ travels from node } i \text{ to node } j \\ 0 \ , & \text{otherwise} \end{cases}$$

Mathematical Formulation

$$\sum_{j \in N} x_{hj}^k \le 1 \qquad \forall k \in V \ , \ \forall h \in O \tag{1}$$

$$\sum_{i \in N} x_{ih}^k \le 1 \qquad \forall k \in V \ , \ \forall h \in O \tag{2}$$

$$\sum_{i \in N \cup O} \sum_{k \in V} x_{ij}^k = 1 \qquad \forall j \in N \tag{3}$$

$$\sum_{j \in N \cup O} \sum_{k \in V} x_{ij}^k = 1 \qquad \forall i \in N \tag{4}$$

$$x_{ii}^k = 0 \qquad \forall i \in N \cup O, \quad \forall k \in V \tag{5}$$

$$x_{ij}^k + x_{ji}^k \le 1 \quad \forall i, \ j \in N \cup O \ , \forall k \in V \tag{6}$$

$$\sum_{i \in P} q_i = \sum_{i \in D} q_i \tag{7}$$

$$\sum_{\substack{i \in P \\ j \in P \cup \{o\}}} q_i x_{ij}^k \le Q_p \qquad \forall k \in V_p \tag{8}$$

$$\sum_{\substack{i\in D\\ j\in D\cup\{o'\}}} q_i x_{ij}^k \le Q_d \qquad \forall\, k\in V_d \tag{9}$$

$$m=\sum_{k\in V_p}\sum_{j\in P} x_{oj}^k \tag{10}$$

$$m'=\sum_{k\in V_d}\sum_{j\in D} x_{o'j}^k \tag{11}$$

$$ST_j^k = A_t + B_t\, q_j x_{ij}^k \qquad \forall\, i\in N\cup O,\ \forall\, j\in N,\ \forall k\in V \tag{12}$$

$$ST_h^k = A_t + B_t \sum_{\substack{i\in N\\ j\in N\cup O}} q_i x_{ij}^k \qquad \forall k\in V\ ,\ \forall h\in O \tag{13}$$

$$DT_j^k = \left(DT_i^k + tt_{ij} + ST_j^k\right)x_{ij}^k \qquad \forall\, i\in N\cup O,\ \forall\, j\in N,\ \forall k\in V \tag{14}$$

$$AT_j^k = \left(tt_{ij} + DT_i^k\right)x_{ij}^k \qquad \forall\, i\in N\cup O,\ \forall\, j\in N,\ \forall k\in V \tag{15}$$

$$a_i \le AT_i^k \le b_i \qquad \forall\, i\in N,\ \forall k\in V \tag{16}$$

$$AT_{o'}^k = AT_o^k + ST_o^k + \sum_{\substack{i\in P\\ j\in P\cup\{o\}}} q_i x_{ij}^k \qquad \text{and} \qquad RT_{o'} = \max_{k\in V}\left\{AT_{o'}^k\right\} \tag{17}$$

$$DT_{o'}^k = RT_{o'} + ST_{o'}^k \qquad \forall k\in V \tag{18}$$

$$\max_{k\in V}\left\{AT_{o'}^k\right\} \le T \tag{19}$$

$$x_{ij}^k = \{0\ ,1\} \qquad \forall\, i,\ j\in N\cup O,\quad \forall k\in V \tag{20}$$

$$tc_{ij}x_{ij}^k \quad \forall\, i,\ j\in N\cup O,\quad \forall k\in V \tag{21}$$

$$SC_j^k = A_c + B_c\, q_j x_{ij}^k \qquad \forall\, i\in N\cup O,\ \forall\, j\in N,\ \forall k\in V \tag{22}$$

$$SC_h^k = A_c + B_c \sum_{\substack{i\in N\\ j\in N\cup O}} q_i x_{ij}^k \qquad \forall k\in V\ ,\ \forall h\in O \tag{23}$$

$$MC^k = \sum_{k\in V_p}\sum_{\substack{i\in P\\ j\in P\cup\{o\}}} q_i x_{ij}^k \tag{24}$$

$$oc^k x_{hj}^k \quad \forall h\in O\ ,\ \forall\ j\in N\ ,\quad \forall k\in V \tag{25}$$

At the beginning of the routes, all vehicles leave from CDC to nodes and at the end of their routes, they arrive at CDC from nodes which are represented by (1) and (2) respectively. Equations (3) and (4) indicate that, one vehicle has to arrive at one node and to leave at one node respectively. Loops in routes and backward movement routes are prevented by (5) and (6) respectively. The equilibrium condition is shown in (7). Capacity constraints of inbound and outbound vehicles are given by (8) and (9) respectively. Equations (10) and (11) expose the required number of inbound and outbound vehicles respectively.

The service time at a node and at the doors of CDC are represented by (12) and (13) respectively. Equations (14) and (15) indicate the departure and arrival times of nodes. Time window of a node is shown in (16). Equations in (17) expose both arrival time and the ready time at outdoors of CDC for reloading quantity to outbound vehicles. Departure time of outbound vehicles is measured by (18). Inequality (19) shows the time planning horizon. The binary integer values of the decision variables are defined in (20).

The components of total cost are calculated as follows. The transportation cost between nodes is noted by (21). The service costs at a node and at the doors of CDC are measured by (22) and (23) respectively. Equation (24) is used to determine the cost of moving shipments inside CDC. The vehicle operational cost is found using (25).

The *objective function* is to minimize the total cost (TC) which includes transportation cost between pickup nodes and delivery nodes, service cost at each node, service cost at CDC, shipping cost from indoors to outdoors of CDC, and vehicle operational cost which is formulated as:

$$MinTC = \left(\sum_{k\in V} \sum_{i,j\in N\cup O} tc_{ij} x_{ij}^{k} \right) + \left(\sum_{k\in V} \sum_{\substack{i\in N\cup O \\ j\in N}} SC_{j}^{k} x_{ij}^{k} \right) + \left(\sum_{k\in V} \sum_{\substack{i\in N \\ h\in O}} SC_{h}^{k} x_{ih}^{k} \right)$$

$$+ \left(\sum_{k\in V_p} \sum_{\substack{i\in P \\ j\in P\cup\{o\}}} q_i x_{ij}^{k} \right) + \left(\sum_{k\in V} \sum_{\substack{j\in N \\ h\in O}} oc^{k} x_{hj}^{k} \right)$$

Method of Solution

A Mixed Integer Non-Linear Programming (MINLP) model is developed to solve TW-VRPCD-MS problem. The problem is coded in LINGO (version 18) optimization software and solved using Branch and Bound (BB) algorithm. These programmes were run on Intel Core i5 with 2.30 GHz CPU and 4 GB RAM. The feasibility of the proposed model is tested using small-scale of TW-VRPCD-MS randomly generated instances as described in Table 1 given below:

Table 1. *Parameter values*

Input	Distribution/ Value	Input	Distribution/ Value
Travelling time	Uniform (20, 100)	Travelling cost	Uniform (50, 200)
Quantity	Uniform (20, 50)	Time horizon	960 minutes
Preparation time	10 units (minutes)	Unit shipping time	1 unit (minute)
Preparation cost	10 units (currencies)	Unit shipping cost	1 unit (currency)
Inbound vehicle capacity	80	Outbound vehicle capacity	50
Operational		Operational	

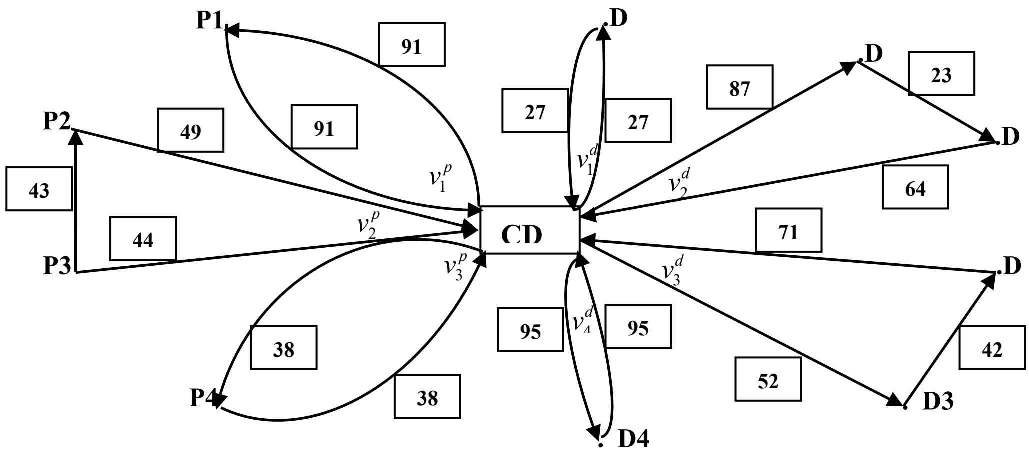
cost of inbound vehicle	150	cost of outbound vehicle	100

Results and Discussion

Results of an instance with 4-suppliers and 6- customers in TW-VRPCD-MS

In this subsection, the computations of relevant times are illustrated with an instance consist of 4- suppliers and 6- customers in the proposed model TW-VRPCD-MS.

Figure 3 illustrates the optimal solution routes of the instance with 4- suppliers and 6- customers in the model TW-VRPCD-MS. In the pickup process to collect the single commodity products from 4- suppliers, only 3- inbound vehicles are needed (v_1^p, v_2^p and v_3^p), where as in the delivery process to distribute those products to 6- suppliers, 4- outbound vehicles (v_1^d, v_2^d , v_3^d and v_4^d) are needed.



P- pickup node; D- delivery node; tt - traveling time;
v^p - inbound vehicle v^d - outbound vehicle:

Figure 3. Solution of the instance with 4- suppliers and 6- customers in TW-VRPCD-MS

The summary of the results obtained for the instance with 4- suppliers and 6- customers in the model TW-VRPCD-MS are presented in Table 2. The input values (q_i, a_i and b_i) of the model are reported from 4th to 6th columns. The output values from the model are included from 7th to 12th columns in Table 2. Thus, the arrival time of the pickup node P2 is calculated according to the equation (15) as follows:

$AT_{P2}^{v_2^p} = \left(tt_{32} + DT_{P3}^{v_2^p}\right) x_{32}^{v_2^p} = \left(43+89\right)\times 1 = 132\,\text{min}.$ Though the travelling time between CDC and supplier P4 is $tt_{o4} = 38$ min, due to the lower bound a_i of the time window of the pickup node P4 is 120m, v_3^p cannot serve P4 before 120 min and thereby has to wait till 120 min. Therefore, the arrival time of P4 is $AT_{P4}^{v_3^p} = 120$ min. The service time at the supplier P3 is obviously measured using (12) and accordingly, $ST_{P3}^{v_2^p} = 10 + 35 = 45$ min. Also, the departure time at the supplier P3 is determined by (14) such as $DT_{p3}^{v_2^p} = AT_{p3}^{v_2^p} + ST_{p3}^{v_2^p} = 44 + 45 = 89$ min.

Once each inbound vehicle arrives at indoors of CDC, it will take 10 min to prepare for unloading and 1 min to unload per pallet and according to the equation (13), service time at indoors of CDC are calculated as $ST_o^{v_2^p} = 10 + 72 = 82$ min. Soon after unloading products, they will be shipped near the outdoors of CDC and the moving time (MT) is calculated as $\sum_{\substack{i \in P \\ j \in P \cup \{o\}}} q_i x_{ij}^k$.

Therefore, $MT^{v_2^p} = 72$ min. The arrival time of the products, collected by inbound vehicle v_2^p, near the outdoors of CDC is determined by (17), as $AT_{o'}^{v_2^p} = AT_o^{v_2^p} + ST_o^{v_2^p} + \sum_{\substack{i \in P \\ j \in P \cup \{o\}}} q_i x_{ij}^{v_2^p} = 228 + [10 + 72] + 72 = 382$ min.

Thus the ready time (RT) at the outdoors of CDC is determined by again (17) as RT= max {231 + 88, 228 + 154, 07 + 88 } = 382 min.

Table 2. *Results of the instance with 4- suppliers and 6- customers in TW-VRPCD-MS*

Process	Vehicle	Node	q_i	a_i	b_i	AT_i^k	ST_i^k	DT_i^k	AT_o^k	$\sum_k q_i$	$AT_{o'}^k$
Pickup	v_1^p	P1	39	60	120	91	49	140	231	39	319
	v_2^p	P3	35	30	90	44	45	89	228	35+37=	**382**
		P2	37	90	150	132	47	179		72	
	v_3^p	P4	39	120	180	120	49	169	207	39	358
Delivery	v_1^d	D1	27	450	480	468	37	505	532	27	
	v_2^d	D2	28	510	540	528	38	566	**750**	28+21=	
		D5	21	600	720	648	31	679		49	
	v_3^d	D3	23	420	540	493	33	526	655	23+22=	
		D6	22	540	570	559	32	591		45	
	v_4^d	D4	29	510	540	536	39	575	670	29	

Furthermore, the finish time (FT) of the entire process is calculated as per the equation (19) to report that the time horizon constraint is satisfied and it is FT = max {532, 750 , 655 , 670} = 750 min which is less than 960 min. In addition, RT and FT are highlighted in bold under the columns $AT_{o'}^k$ and AT_o^k respectively in Table 2. All these information of the instance with 4- suppliers and 6- customers are summarized in Table 2 above:

Computational Experiments of TW-VRPCD-MS

Table 3 summarizes the results of sixteen small-scale instances of the model TW-VRPCD-MS. As the input data to the model, the number of pickup and delivery nodes, the total flow of the network are included from 2nd to 4th columns of Table 3. Accordingly, the output obtained from the LINGO such as required number of inbound and outbound vehicles, the optimal solution in terms of cost and the average computational time of 10 replicates of the same instance are included from 5th to 8th columns in Table 3:

Table 3. *Results of small-scale instances of TW-VRPCD-MS*

Instance No.	**No. of Nodes**		**Flow**	**No. of Vehicles Used**		**Optimal Solution**	**Average Computational Time T (in s)**
	Pickup	Delivery		Inbound	Outbound		
01	02	03	100	02	03	2606	0.132
02	03	03	110	03	03	3186	0.164
03	03	04	120	03	04	3520	0.197
04	03	05	130	03	05	3806	0.240
05	04	05	140	02	05	3708	0.364
06	**04**	**06**	**150**	**03**	**04**	**3887**	**0.750**
07	04	07	160	03	04	4002	1.534
08	05	07	170	03	04	4120	3.123
09	06	07	180	03	06	4603	5.566
10	06	08	190	05	04	5063	11.591
11	07	08	200	03	05	4917	25.916
12	08	08	210	03	07	5733	66.355
13	08	09	220	03	05	4618	135.589
14	08	10	230	05	05	5589	316.818
15	09	10	240	03	05	5343	410.835
16	10	10	250	04	06	5683	661.932

The applicability of the proposed MINLP model could be observed from the results of the computational experiments exhibited in Table 3. To explain in details of the same problem (highlighted in bold in Table 3) discussed in the above subsection 5.1, the 6th instance is considered here as well. To collect 150 units of products from 4-suppliers, 3- inbound vehicles are used whilst 4-outbound vehicles are needed to distribute those 150 units of products to 6- customers. The factors of the total cost (TC), to complete the entire process while satisfying all required time related constraints, are determined as follows:

Table 4. *Factors of total cost in TW-VRPCD-MS*

Factors of total cost	Costs at pickup process	Costs at delivery process
Transportation cost	1051	1066
Service cost at nodes	190	210
Service cost at CDC	180	190
Moving Shipments cost	150	N/A
Vehicle operational cost	450	400

Total cost (TC), as described in the objective function of the model in the subsection 4.2, is **Rs 3887.00** and this optimal solution can be obtained through LINGO in **0.75 sec** as the computational time. Moreover, since the average computational time is reasonably less for the above instances in Table 3, this model can be used for last time planning for small-scale problems of TW-VRPCD-MS with nodes up to 20.

Convergence Analysis

This subsection analyses the rate of convergence. Data for this analysis are extracted from Table 3. The plot of the average computational time T (in the last column in Table 3), against the total number, x (sum of the values of columns 2nd and 3rd in Table 3), of the suppliers and customers as instance size is presented in Figure 4. It can be obtained from the fitted curve in Figure 4, that the average computational time to obtain the optimal solution increases according to the exponential function, $T(x) = 0.002e^{0.63x}$. Furthermore, as the goodness of fitted curve, the coefficient of determinant, R-squared value, is obtained as 98%. Hence, it can be stated that, the convergence rate of the problems considered in this study rises to exponential.

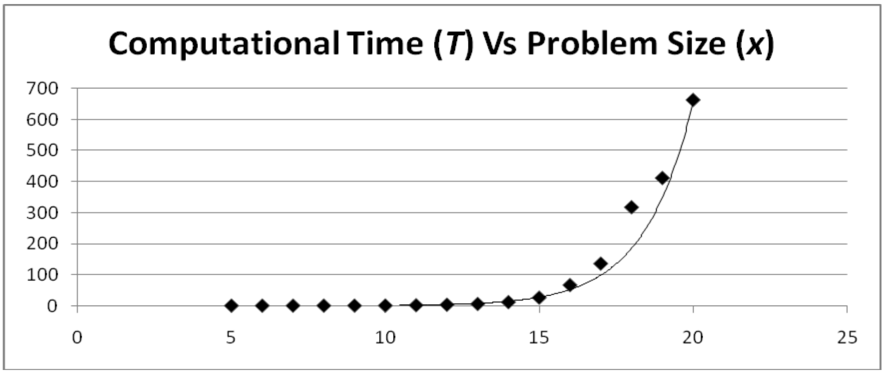


Figure 4. Plot of Average Computational Time Vs Problem Size

It is to be emphasized that, almost all the problems where the input size exceeds 20 nodes, a feasible solution could be obtained with a CPU time less than 10 seconds. Therefore, this study suggests that, to solve medium and moderately large-scale instances, heuristic or meta-heuristic algorithms are more appropriate to obtain a near optimal solution in a reasonable computational time.

Conclusions and Recommendations

A mixed integer nonlinear programming model is developed to obtain the optimal solutions to hard time windows vehicle routing problem with moving shipments at the cross-dock centre (TW-VRPCD-MS). In addition to the time windows aspect, moving shipments, an activity inside a cross-dock centre is mainly considered in this study. Since the average computational time is reasonably less for the instances considered in this study, it could be concluded that this proposed model could be used for last time planning for similar size instances with nodes up to 20. Moreover, the convergence rate of the instances considered in this study is exponential. Thus, it could be concluded that when the number of nodes increases, consequently the computational time to reach the optimal solution increases exponentially. Therefore, this study recommends that heuristic or meta-heuristic methods are more appropriate to solve the medium scale problems with nodes between 20 and 50 and large-scale problems with nodes more than 50 of TW-VRPCD-MS to obtain a near optimal solution in a moderately small computational time. Moreover, it is recommended for further studies to revise the proposed model according to the availability of vehicles for transportation, temporary storage capacity at CDC and budget allocation for the transportation.

Conflict of Interest

The authors declare that there are no known conflicts of interest associated with the content of this article.

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