DOI: https://doi.org/10.4038/ouslj.v18i2.7622

OUSL Journal, 2023
Vol. 18, No. 2, (pp 87-106)

Optimal Solution for the Integrated Model of the Problems in Capacitated Vehicle Routing and Vehicle Scheduling at the Multi-door Depot

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Abstract

To have an efficient Supply Chain (*SC*), the coordination and integration of the activities in the *SC* are mandatory. Routing vehicles is an optimization problem in the *SC*. When the routed vehicles return to the depot and the doors at the depot are busy, the returned vehicles must wait to unload the accumulated shipments. Therefore, properly scheduling these vehicles to those doors at the depot to minimize the waiting time is an optimization problem in the *SC*. Therefore, in this study, *routing vehicles* to collect the shipments from suppliers and *scheduling vehicles* to doors at the depot, based on a *first-come-first-serve* basis, are simultaneously solved. Hence, the objective of this integrated vehicle routing and scheduling problem (*VR&SP*) is to minimize the total cost which contains the following components: *vehicle travelling cost* between suppliers,

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(Received 11th September 2023; Revised 20th December 2023; Accepted 31st December 2023) © OUSL)

loading cost at the suppliers, *vehicle waiting cost*, *unloading cost* at the depot and *vehicle operations cost*. A Mixed Integer Quadratic Programming (*MIQP*) model is developed to solve the integrated *VR&SP*. The *Branch and Bound* algorithm method is employed to obtain the exact optimal solution to this *MIQP* using *LINGO* optimization software. The compatibility of the developed *MIQP* model is verified by the randomly generated small-scale instances of *VR&SP*. Therefore, it can be concluded that, this model solves the vehicle routing to suppliers and vehicle scheduling to doors at the depot simultaneously. Since *VR&SP* is a *NP*-hard problem, heuristics or meta-heuristic methods are recommended to solve the large-scale instances of *VR&SP*. Moreover, it can be recommended for further studies to amend this model by incorporating additional constraints to make it more applicable to real-world scenarios.

Keywords: multi-door depot, optimization, vehicle routing, vehicle scheduling

Introduction

The significance of the supply chain plays an important role in the competitive environment of the global market. The goal of the efficient supply chain is that the shipments must be supplied and delivered to the right place in the right quantity at the right time with a low cost. Therefore, to attain this goal, the proper coordination of the activities in the supply chain is mandatory. Vehicle Routing Problem (VRP) is one of the main combinatorial optimization problems in the supply chain. The optimal scheduling of the VRP is one of the influential factors of the efficiency of a supply chain. The VRP was introduced in 1959 by Dantzig and Ramser (Dantzig & Ramser, 1959). It has several variants based on its characteristics and Capacitated Vehicle Routing Problem (CVRP) is one of them (see, e.g., (Toth & Vigo, 2002)). The CVRP consists of determining vehicle routes through a set of geographically scattered customers, subject to the various constraints including the limitation of the capacity of the vehicles. The common objective of CVRP is to minimize the transportation cost in terms of travel distance or travel time. In this study, the closed network configuration of CVRP is taken into consideration. Therefore, after collecting shipments from the suppliers, vehicles must return to the depot at the end of their routes.

Once the shipments are collected from the manufacturers or suppliers, the routed vehicles must return to the collection center and these vehicles may arrive at different times. Usually this center is called a 'depot'. A depot can have different layouts in terms of its number of doors. On the one hand, the basic layout is that depot having a "single door". In this case, scheduling is a matter of sequencing vehicles to the door of the depot. On the other hand, the depot can have multiple doors to receive the collected shipments from the routed vehicles. In the later case of layout, scheduling become not only sequencing vehicles to a selected door, but also assigning them to the doors of the depot. Consequently, the scheduling in the second case is a much more complicated problem than the single door layout case. Since these doors are scarce resource, properly scheduling these vehicles to those doors at the depot is another key issue in the supply chain. The Vehicle Scheduling Problem (VSP) leads to assigning the vehicles to doors at the depot and sequencing them to each door, to minimize the waiting time, to unload the shipments, which causes the additional cost and increases the total cost.

In the literature, regarding the supply chain, revealed that most of the studies focused only on one problem, but it is recommended to deal with several problems together (Van Belle et al., 2012) and to include some of the operations at the depot with the vehicle routing problem (Buakum & Wisittipanich, 2019). Therefore, in this study, routing the vehicles to collect the shipments from suppliers and scheduling the routed vehicles to doors at the depot are simultaneously solved. The integrated problem of VRP and

VSP is referred to in this study as *VR&SP*. Hence, the objective of this integrated *VR&SP* is to minimize the total cost which contains the following components: *vehicle travelling cost* between suppliers including depot, *loading cost* at the suppliers, and *vehicle waiting cost*, *unloading cost* at the depot and *vehicle operations cost*. It should be emphasized that the total cost consists not only the vehicle routing and operational costs, but also the cost due to vehicle waiting at the parking yard of the depot. To the best of authors' knowledge, there is no study found in the literature that integrates VRP and VSP. Moreover, not only the vehicle routing problem (Lenstra, J. K.; Rinnooy Kan, 1981), but also the vehicle scheduling problem is classified as a *NP*-hard problem (Boysen et al., 2010). Hence the integrated *VR&SP* is also obviously a *NP*-hard problem. Therefore, only the small-scale instances are randomly generated to obtain exact optimal solution to the *VR&SP* problem. Since this study contributes a novel idea of integrating VRP and VSP, it can be upgraded to real-world supply chain scenarios by incorporating relevant constraints.

It is emphasized that this work already has been presented in a research conference and this full paper is the expanded version of the same research. The outline of this paper is as follows: Section 2 reviews the previous studies on the integrated model. The problem under investigation is explained in Section 3. In Section 4, the details of the model development and solution method are given. The findings of randomly generated small-scale instances are discussed in Section 5. Section 6 summarizes the findings based on the outcome from this study and recommendations for further research.

Review of literature

In the research on supply chain management, the Vehicle Routing Problem (VRP) has been studied extensively after the study by Danzig and Ramser (Dantzig & Ramser, 1959).

So far many variants of VRP have been formulated and Capacitated VRP (CVRP) is a basic and well-studied problem in the literature (Toth & Vigo, 2002). The objective of CVRP is minimizing the total cost with specified constraints which includes the limited capacity of the vehicles. To a significant performance of a supply chain, the coordination among the components of supply chain must be efficient. Therefore, the VRP is integrated with some other problems involved in the components of the supply chain. For instance, machine scheduling was integrated with VRP to minimize total carbon emission (Wang et al., 2019), shift scheduling was integrated with VRP in waste collection (De Bruecker et al., 2018) and production scheduling was integrated with VRP at the operational level decision (Moons et al., 2017). To the best of authors' knowledge, the existing literature on VRP has not jointly considered Vehicle Routing Problem and Vehicle Scheduling Problem together. This concept is mainly motivated by the recommendations made by the literature surveys (Van Belle et al., 2012) and (Buakum & Wisittipanich, 2019).

The layout of a depot in a distribution network can have only a single door to unload the accumulated shipments or it can have multi-doors to unload the shipments simultaneously. In the single door case, scheduling the vehicles to the door is only a matter of sequencing them to that particular door. However, in multi-door depot case, scheduling become first assigning a door to a vehicle and then sequencing vehicles those who are assigned to that particular door. This study considers the second case in which the depot has multi-doors to unload the accumulated shipments concurrently. There has been little research on truck scheduling to the doors at the depot in a Vehicle Routing Problem. However, there are plenty of researches on the vehicle scheduling with the cross-docking centers since the initial study (Li et al., 2004) in 2004. In (Li et al., 2004), vehicle scheduling problem was modeled as machine scheduling problem with just-in-time concept. A single in-door depot was considered in (Yu & Egbelu, 2008) to schedule the vehicles in order to minimize the makespan at the cross-docking center. Later in 2010, Boysen started studying the vehicle scheduling with multi-door cross-docking center (Boysen, 2010). Consequently, in this study, Vehicle Routing Problem and Vehicle Scheduling Problem are solved simultaneously and the integrated problem is described in the next section.

The sequence of the integrated *VR&SP*

The sequence of the process of the integrated *VR&SP* is described in this section. The entire process is divided into two phases which are briefly explained as follows:

At the first phase of the integrated *VR&SP*, only the Capacitated Vehicle Routing Problem (CVRP) is considered. Accordingly, the homogeneous vehicles (capacities of vehicles are equal) initiate their routes from the depot at time zero and visit all assigned suppliers. After loading all the shipments at the suppliers in the allotted routes, all the vehicles return to the depot and wait (at the parking yard of the depot) until the turn to come to unload the shipments through the doors at the depot. It should be emphasized that, not only the cost components relevant to routing vehicles due to travelling between suppliers, loading shipments at suppliers; but also, the time components such as *travelling time* and *loading time* are also calculated in each route. In addition to those time components, the arrival time of each vehicle to the depot is recorded. This arrival time of each vehicle will be employed to calculate the 'waiting time' in the second phase. Moreover, the cost for the routing vehicles and operations cost are added to the total cost of the integrated *VR&SP* problem. Figure 1 depicts the process of CVRP.

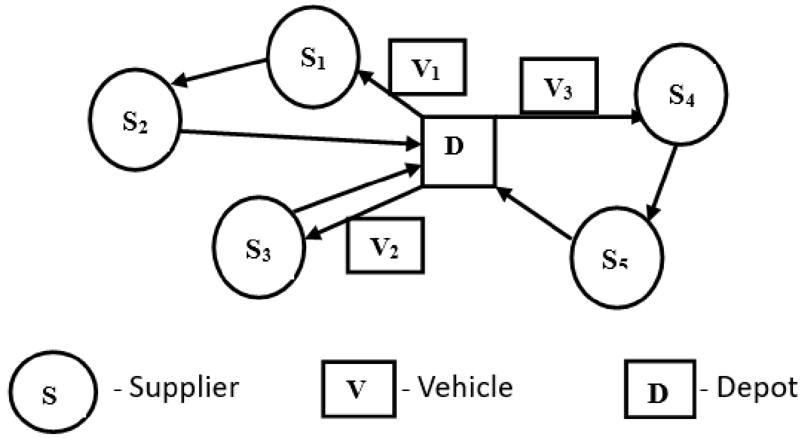

Figure 1: *Process of Capacitated Vehicle Routing Problem*

At the second phase of the integrated *VR&SP*, only the Vehicle Scheduling Problem (VSP) is considered. Based on the arrival time, the vehicles which arrive at the parking yard of the depot are assigned to the doors under the first come first serve policy. At the same time, the sequencing of the assigned vehicles to each door at the depot is taken place. In this second phase, in addition to the cost components relevant to scheduling vehicles due to vehicle waiting, vehicle changeover and unloading; vehicle *waiting time* at the parking yard, *vehicle changeover time* to the doors at the depot and accumulated shipments *unloading time* at the doors of the depot are also determined. Then the waiting time of the vehicles is converted into cost with a pre-defined conversion ratio. Consequently, all the aforementioned costs relevant to the second phase are added to the total cost of the integrated *VR&SP* problem. Figure 2 portrays the process of VSP.

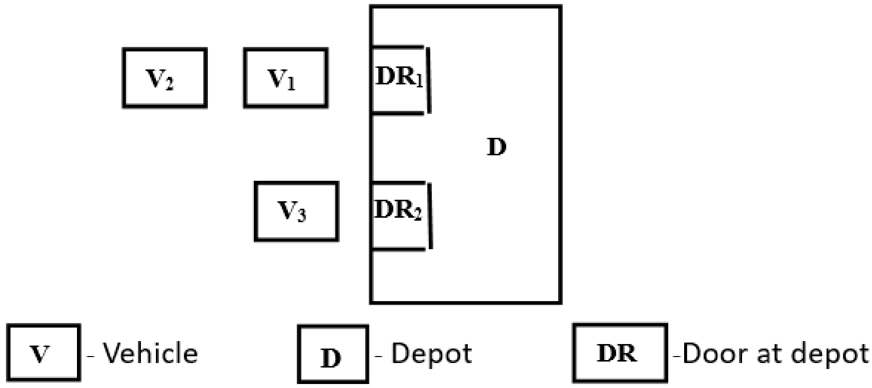

Figure 2: *Process of Vehicle Scheduling Problem*

Eventually, the sum of all the components of the total cost in both phases is considered as the solution to the integrated *VR&SP* problem.

The assumptions of the model

In this study, the following characteristics are assumed when developing the model for the integrated VR&VP:

- Single depot with multi doors
- Capacitated Vehicle Routing Problem with homogeneous fleets of vehicles
- Closed Vehicle Routing Problem and vehicles are available at time zero
- Split pickup is not allowed, and each supplier must be served by exactly one vehicle
- Vehicles changeover time at the doors of the depot is fixed
- Each door processes one vehicle at a time and is assumed to be sufficiently equipped with equipments (hand stackers or forklifts) and laborers

Since this study of integrating two *NP*-hard problems VRP and VSP, which is highly complicated and new to the literature, these characteristics must be assumed initially. Also, some factors such as: varying demand, dynamic scheduling, and unforeseen delays are presupposed deterministic. Therefore, the basic variant CVRP is integrated with VSP, and small-scale instances are taken for numerical experiments. However, other variants of VRP such as Open VRP, VRP with Time Windows and Green VRP can be attempted in further studies. In those future works, aspects like cost, time, and resource requirements can be implemented to the real-world setting.

Components of the total cost of the integrated *VR&SP*

The total cost incurred due to the following operations taking place at the integrated *VR&SP* in this study:

- *Vehicle travelling cost* not only in-between suppliers but also between depot and suppliers
- *Loading cost* of the shipments at the suppliers
- *Vehicle waiting cost* at the parking yard of the depot before unloading their shipments at the doors of the depot
- *Unloading cost* of the accumulated shipments (from suppliers) at the doors of the depot
- *Vehicle operations cost* to maintain the vehicles by the depot

Model Formulation

Notations

Indices

i, j : Indices for suppliers

h : Index for doors of the depot

k, l : Indices for vehicles

General parameters

n : Number of suppliers designated to the depot

m : Number of homogeneous vehicles available at the depot

r : Number of doors at the depot

q_i : Shipments at supplier i

Q : Maximum capacity of homogeneous vehicles

M : A big number

Cost parameters

TC_{ij} : Travelling cost from supplier i to supplier j

OC^k : Operational cost of vehicle k

LC_i^k : Loading cost at supplier i by vehicle k

UC_h^k : Unloading cost at door h by vehicle k

A_c : Fixed cost of preparation for loading shipments at suppliers or unloading shipments at doors at the depot

B_c : Variable cost of loading a unit of shipments at suppliers or unloading a unit of shipments at doors at the depot

Time parameters

TT_{ij} : Travelling time from supplier i to supplier j

LT_i^k : Loading time at supplier i by vehicle k

A_t : Fixed time of preparation for loading shipments at suppliers

B_t : Variable time of loading a unit of shipments at suppliers or unloading a unit of shipments at doors at the depot

CT^k : Fixed vehicle changeover time

AT_i^k : Arrival time of vehicle k at supplier i

DT_i^k : Departure of vehicle k at supplier i

PT^k : Processing time of vehicle k at doors of depot

BT_h^k : Beginning time of vehicle k to unload at door h of depot

ET_h^k : Ending time of vehicle k after unloading at door h of depot

WT_h^k : Waiting time of vehicle k to unload at the door h of depot

Sets

$S = \{S_1, S_2, ..., S_n\}$: Set of n suppliers

$V = \{v_1, v \ , ..., v_m\}$: Set of m vehicles

$O = \{o_1, o_2, ..., o_r\}$: Set of r doors of the depot

Binary Variables

$$x_{ij}^k = \begin{cases} 1, & \text{if vehicle } k \text{ travels from supplier } i \text{ to supplier } j \\ 0, & \text{otherwise} \end{cases}$$

$$x_h^k = \begin{cases} 1 , & \text{if vehicle } k \text{ assigns to door } h \\ 0 , & \text{otherwise} \end{cases}$$

$$z_h^{kl} = \begin{cases} 1 , & \text{if vehicle } k \text{ precedes vehicle } l \text{ in the vehicle sequence at door } h \\ & \text{when } k \neq l \text{ or if vehicle } k \text{ is the first vehicle in door } h \text{ when } k = l \\ 0 , & \text{otherwise} \end{cases}$$

Mathematical Formulation

The *objective function* of the *VR&SP* is to minimize the total cost incurred by the travelling between suppliers, loading at each supplier, unloading at depot, vehicle operations, vehicle waiting which is formulated as:

$$Minizing\ Total\ Cost = (Travelling\ Cost) + (Loading\ Cost) + (Waiting\ Cost) + (Unloading\ Cost) + (Operations\ Cost)$$

The components of the total cost are defined in the following sub-section.

Determination of the components of the total cost

$$Total\ Travelling\ Cost = \sum_{k \in V} \sum_{i, j \in S \cup O} TC_{ij} x_{ij}^k$$

$$Total\ Loading\ Cost = \sum_{k \in V} \sum_{\substack{i \in S \cup O \\ j \in S}} A_c + B_c\, q_j x_{ij}^k$$

$$Total\ Unloading\ Cost = \sum_{k \in V} \left[A_c + B_c \sum_{\substack{i \in S \\ h \in O}} q_i x_{ih}^k \right]$$

$$Total\ Waiting\ Cost = \sum_{k \in V} \sum_{\substack{i \in S \cup O \\ h \in O}} \left[BT_h^k - AT_{ih}^k \right]$$

$$Vehicle\,Operations\,Cost = OC^k \sum_{k \in V} \sum_{\substack{j \in S \\ h \in O}} x_{hj}^k$$

Subject to the following constraints to the integrated *VR&SP*:

Vehicle routing constraints

$$\sum_{j \in S} x_{hj}^k \le 1 \qquad \forall k \in V\ ,\ \forall h \in O \tag{1}$$

$$\sum_{i \in S} x_{ih}^k \le 1 \qquad \forall k \in V\ ,\ \forall h \in O \tag{2}$$

$$\sum_{i \in S \cup O} \sum_{k \in V} x_{ij}^k = 1 \qquad \forall j \in S \tag{3}$$

$$\sum_{j \in S \cup O} \sum_{k \in V} x_{ij}^k = 1 \qquad \forall i \in S \tag{4}$$

$$x_{ii}^k = 0 \qquad \forall i \in S \cup O,\quad \forall k \in V \tag{5}$$

$$x_{ij}^k + x_{ji}^k \le 1 \quad \forall i,\ j \in S \cup O\ ,\ \forall k \in V \tag{6}$$

$$\sum_{\substack{i \in S \\ j \in S \cup \{o\}}} q_i x_{ij}^k \le Q \qquad \forall k \in V \tag{7}$$

Inequalities (1) and (2) respectively represent that all the vehicles begin their routes by departing the depot to designated suppliers and arrive at the depot at the end of their routes. Only a single vehicle must arrive at a supplier and depart from a supplier is indicated in the equations (3) and (4) respectively. While loops in any route are prevented by (5), backward movements in routes are prevented by (6). Also (7) states that the accumulated shipments from suppliers cannot be exceeded the capacity of a vehicle at any time.

Time calculations at the suppliers when routing vehicles

$$LT_j^k = A_t + B_t\, q_j x_{ij}^k \qquad \forall i \in S \cup O,\ \forall j \in S,\ \forall k \in V \tag{8}$$

$$DT_j^k = \left(DT_i^k + TT_{ij} + LT_j^k \right) x_{ij}^k \qquad \forall i \in S \cup O,\ \forall j \in S,\ \forall k \in V \tag{9}$$

$$AT_j^k = \left(TT_{ij} + DT_i^k\right)x_{ij}^k \qquad \forall i \in S \cup O,\ \forall j \in S,\ \forall k \in V \qquad (10)$$

At the suppliers, the loading time, departure time and arrival time are respectively calculated according to the equations (8), (9) and (10) given above.

Vehicle Scheduling Constraints

$$\sum_{h \in O} x_h^k = 1 \quad \forall k \in V \qquad (11)$$

$$x_h^k + x_h^l - 1 \le z_h^{kl} + z_h^{lk} \quad \forall k, l \in V,\ k \ne l,\ \forall h \in O \qquad (12)$$

$$z_h^{kl} + z_h^{lk} \le 1 \quad \forall k, l \in V,\ \forall h \in O \qquad (13)$$

The equation (11) ensures that every vehicle is assigned to a single door. The relationship between the variables x_h^k, x_h^l and z_h^{kl} for the vehicles is indicated by (12). The inequality (13) prevents the condition that either vehicle k precedes vehicle l or vehicle l precedes vehicle k at the door h.

Time calculations at the doors of the depot when scheduling vehicles

$$BT_h^k \ge AT_{ih}^k \quad \forall k \in V,\ \forall h \in O \qquad (14)$$

$$PT_h^k \ge CT^k + B_t \sum_{i \in S \cup O} q_i x_{ih}^k \quad \forall k \in V,\ \forall h \in O \qquad (15)$$

$$ET_h^k \ge AT_{ih}^k + PT_h^k \quad \forall k \in V,\ \forall h \in O \qquad (16)$$

$$BT_h^l \ge ET_h^k + CT^k - M(1 - z_h^{kl}) \quad \forall k, l \in V,\ \forall h \in O \qquad (17)$$

$$WT_h^k \ge BT_h^k - AT_{ih}^k \qquad \forall k \in V,\ \ \forall h \in O \qquad (18)$$

The starting time of each vehicle is at least the arrival time of the vehicle is ensured by (14). Condition (15) is used to determine the processing time of each vehicle at the doors of the depot. The inequality (16) calculates the end time of each vehicle after the processing at the depot. The end time of each vehicle which predecessors plus vehicle changeover time is shown in (17). Finally, (18) exposes the waiting time of each vehicle at the parking yard of the depot. Also, it is emphasized that all the variables and parameters are assumed here are non-negative.

Data generation and solution method

The Branch and Bound (B&B) algorithm is employed to obtain the exact optimal solution to this *MIQP* using *LINGO* (version 18) optimization software. The programs are run on Intel *core i5* with 2.30 *GHZ CPU* and 4 *GB RAM*. The input data for the instances of *VR&SP* are generated randomly based on the following parameters reported in the Table 1 given below:

Table 1. *Parameters of MIQP model of VR&SP*

Parameters	Distribution/ Value	Parameters	Distribution/ Value
Travelling cost	Uniform (50, 200)	Travelling time	Uniform (20, 100)
Shipment	Uniform (20, 50)	Vehicle capacity	60 units
Unit loading cost	1 cost unit	Unit loading time	1 time unit
Unit unloading cost	1 cost unit	Unit unloading time	1 time unit
Preparation cost	10 cost units	Preparation time	10 time units
Vehicle operations cost	50	Vehicle changeover time	15 time units

Findings and Discussion

Results of small-scale instances of *VR&SP*

Since not only the Vehicle Routing Problem is a *NP*-hard problem (Lenstra, J. K.; Rinnooy Kan, 1981), but also the Vehicle Scheduling Problem is a *NP*-hard problem (Boysen et al., 2010), integrated Vehicle Routing Problem with Vehicle Scheduling Problem is also a *NP*-hard problem. Furthermore, *LINGO* is not capable of handling large-scale instances (as it always tries to obtain the exact optimum solution). Therefore, only the small-scale instances are taken to test the compatibility of the developed *MIQP* for *VR&SP* model. According to the parameters assigned in the

Table 1, fourteen small-scale test instances are generated, and the results of those instances are summarized in the

It should be noted that the size of the instance, in terms of the number of suppliers and number of doors at the depot, gradually increases as exhibited in Table 2. The total shipments that must be collected from the number of appropriate suppliers with the required number of vehicles to the Vehicle Routing Problem are reported in Table 2 given above. In addition, in Table 2, the allocated number of doors at the depot of Vehicle Scheduling Problem is included. Also, Table 2 presents the total waiting time of the vehicles before unloading its shipments and the solutions to the *VR&SP*. As seen from Table 2, the feasibility of the developed *MIQP* model for the *VR&SP* is verified.

Table 2. *Results of small-scale instances of VR&SP*

No. of Suppliers	Total Shipments	Required No. of Vehicles	Allocated No. of Doors	Total Waiting Time (m)	Solution to *VR&SP*
07	200	4	2	11	1,347
08	220	4	2	50	1,627
09	240	5	2	88	1,717
10	260	5	2	183	2,215
11	280	6	3	13	1,952
12	300	6	3	38	2,095
13	320	7	3	95	2,456
14	340	7	4	171	2,795
15	360	7	4	33	2,572
16	380	8	4	26	2,726
17	400	8	5	0	2,703
18	420	8	5	21	2,870
19	440	9	5	41	3,134
20	460	9	5	73	3,306

The results from Table 2 reveal the applicability of the developed *MIQP* for the integrated *VR&SP*. More details of the exact optimal solution of the instance of *VR&SP* with 7-suppliers and 2-doors at the depot (The first instance in the Table 2) are described in the following sub-section:

Results of the instance of *VR&SP* with 7- suppliers and 2-doors

Table 3. *Results of the instance of VR&SP with 7- suppliers and 2-doors*

Vehicle	From	To	Arrival Time	Collected Shipments
V_1	D	S_6	64	32
	S_6	S_2	123	54
	S_2	D	**248**	54
V_2	D	S_3	52	30
	S_3	S_4	136	56
	S_4	D	**267**	56
V_3	D	S_1	27	29
	S_1	S_5	97	57
	S_5	D	**206**	57
V_4	D	S_7	55	33
	S_7	D	**153**	33

This specific instance has 7- Suppliers (S_1 to S_7) and 2-Doors at the depot (DR_1 and DR_2). The route-wise details of this specific instance are illustrated in the Table 3 given above (Here D represents the Depot as described in the Figure 1 and Figure 2):

It can be interpreted from Table 3 given above that the first route by the vehicle V_1 first visits the supplier S_6 from the depot D in 64-time units and collects 32 units of shipments. Next V_1 visits the supplier S_2 from S_6 in 123-time units (in total time from D) and added to 54 units of shipments (it contains the total shipments from both S_6 and S_2). Then V_1 returns to depot D from S_2 in 248 total time units with accumulated 54 units of shipments. Similarly, the detailed results of the routes of the vehicles V_2, V_3 and V_4 can also be interpreted according to the Table 3.

Table 4. *Results of scheduling routed vehicles to doors at the depot*

Door	Vehicle	Accumulated Shipments	Arrival Time (To depot)	Begin Time (To unload)	Processing Time	End Time (To unload)

DR_1	V_4	33	153	153	48	201
DR_1	V_1	54	248	248	69	317
DR_2	V_3	57	206	206	72	278
DR_2	V_2	56	267	278	71	349

According to the "first come first serve" policy and as per the arrival time reported in Table 4, vehicle V_4 (it is the first vehicle to be arrived to the depot D in 153 time units with 33 units of shipments) is assigned to the door DR_1. With the changeover time of 15 time units, 48 (Since, *processing time = changeover time + accumulated shipments*) time units are necessary to completely unload the shipments and the job can be ended in 201 time units. Next, the second vehicle V_3 arrives in 206 time units and it is assigned to door DR_2. Then, the third vehicle V_1 arrives in 248 time units which is assigned to the door DR_1 as it is the only free door at that time. Finally, fourth vehicle V_2 arrives in 267 time units, but both doors are occupied by both vehicles V_3 and V_1 and therefore, it has to wait until the vehicle V_3 ends unloading its shipments at 278 time units (it happens earlier than the finish time of V_1 which is 317 time units). Hence, the total waiting time by all 4 vehicles is 11 (which is equal to 278-267) time units (in fact V_2 is the only vehicle which must wait in this particular instance based on this assignment) and it is converted to 33 cost units (Since it is assumed in this study that 1 unit of waiting time is equivalent to 3 cost units).

Conclusions and Recommendations

In this study, Vehicle Routing Problem and Vehicle Scheduling Problem are solved simultaneously. In the integrated *VR&SP* model, the scheduling routed vehicles from the closed Capacitated Vehicle Routing Problem to the doors of the depot are taken into consideration. A Mixed Integer Quadratic Programming model is developed to solve the *VR&SP* using *LINGO* optimization software. The experimental results of randomly generated 14 small-scale test instances of *VR&SP* show the feasibility of the developed model. Therefore, it can be concluded that this model is suitable for small-scale integrated *VR&SP*. Since *VR&SP* is a *NP*-hard problem, heuristics or meta-heuristic methods are recommended to solve the large-scale instances of *VR&SP*. Furthermore, this *VR&SP* can be extended to Vehicle Routing Problem with cross-docking technique. Generally, cross-docking centers have multi-doors in which some doors are assigned to receiving shipments from suppliers and some are assigned to shipping them to customers. Therefore, it is recommended that an integrated model combining Vehicle Routing Problem and Vehicle Scheduling Problem with cross-docking can be attempted as a future study. Moreover, it can be recommended for further studies to amend this model by incorporating additional constraints to make it more applicable to real world problems. The additional constraints may facilitate the capacity of temporary storage at depot, available vehicles to collect the shipments, budget for components of total cost and uncertainty conditions.

Conflict of Interest

The authors have no conflicts of interest to declare information that is relevant to the content of this article.

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