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Signal Quality Impact of Angular Electromagnetic Wave Reception from a Parabolic Reflector Antenna

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Abstract – *Parabolic reflectors are used in the reception of electromagnetic signals mainly due to two reasons. The first reason is the reflector's ability to precisely focus all the waves incident on the reflector parallel to the main axis at the focal point of the parabola. This implies that the antenna element placed on the focal point can receive all the signals collected by the reflector from a distant point. The second important reason is that all these waves are perfectly in-phase at the focus, so that no destructive interference will take place to weaken the received signal strength at the antenna. Since microwaves have relatively small wave lengths (10 mm – 10 cm), slightest off set in two waves can cause a drastic reduction in the resultant signal due to interference. In this paper it is investigated to what extent the above conditions are satisfied in off-axial waves.*

***Keywords**: angular incidence, parabolic reflector, microwave link, aperture of an antenna*

Nomenclature

VHF - Very High Frequency
UHF - Ultra High Frequency
e.m. waves – Electro Magnetic Waves

1 INTRODUCTION

Use of parabolic reflectors for the reception of microwave- and UHF signals (upper band) is fairly common in the Telecommunication sector. The largest parabolic reflector used in Sri Lanka was the one at Padukka satellite station for the reception of international audio / video signals via INTELSAT satellite system. With the emergence of SEA-ME-WE optical cables, the use of satellite for the international telecommunication was gradually abandoned. Some vendors of the television industry still use parabolic reflectors for the reception of the international TV channels to be broadcast in Sri Lanka. Smaller parabolic dish antennas are mainly used in the cellular communication industry for the establishment of microwave links. In the television industry, a microwave link is normally used to transmit the television signal from the studio to the broadcasting station. The physical parameters and the orientation of the two parabolic reflectors associated with the microwave link can play a vital role in determining picture quality.

2 THEORATICAL BACKGROUND

2.1 Focusing property of a parabolic reflector

The parabolic reflector can be represented by the equation to be $\boldsymbol{y^2 = 4ax}$.

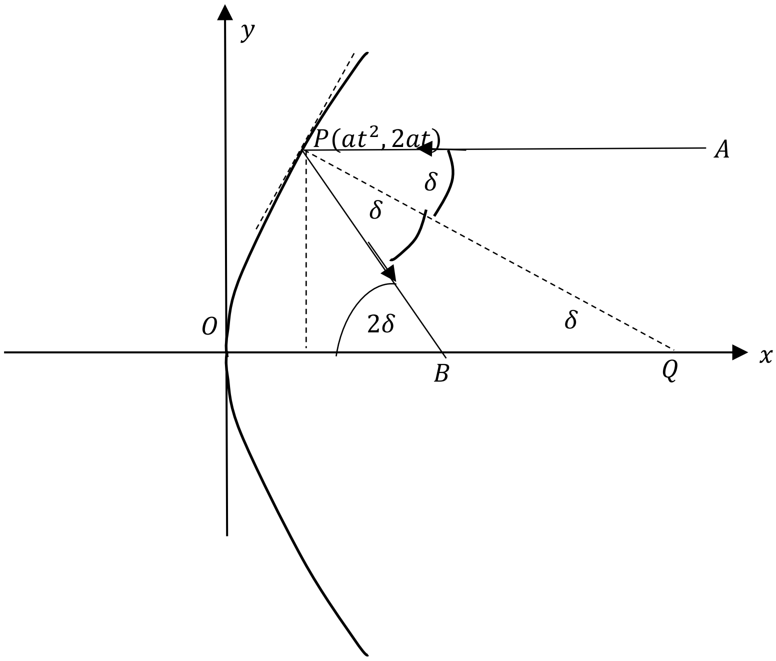

Fig.1: Focusing property of a parabolic reflector

Consider a wave AP parallel to the X-axis is incident on the reflector at a point P. Let AP makes an angle δ with the normal PQ. The reflected ray PB should also make an angle δ with normal PQ according to the rules of reflection.

The gradient of the tangent at P is given by $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} = \frac{2a}{2at} = \frac{1}{t}$

Thus, the gradient of PQ should be $-t$.

Therefore $\tan(\pi - \delta) = -\tan\delta = -t$

The gradient of PB is $\tan(\pi - 2\delta) = -\tan 2\delta = \frac{-2\tan\delta}{1-\tan^2\delta} = \frac{-2t}{1-t^2}$

$\therefore \tan 2\delta = \frac{2at}{OB - at^2} = \frac{2t}{1-t^2} \Rightarrow OB = a$

Thus, we see that all the waves incident on the reflector parallel to the x-axis are focused to the fixed point B.

From this we can conclude that all the waves transmitted in the direction of the receiver by a faraway antenna located on the x-axis of the reflector are collected at the focal point B which is at a distance of a from 0.

2.2 Total phase change of a wave due to reflection

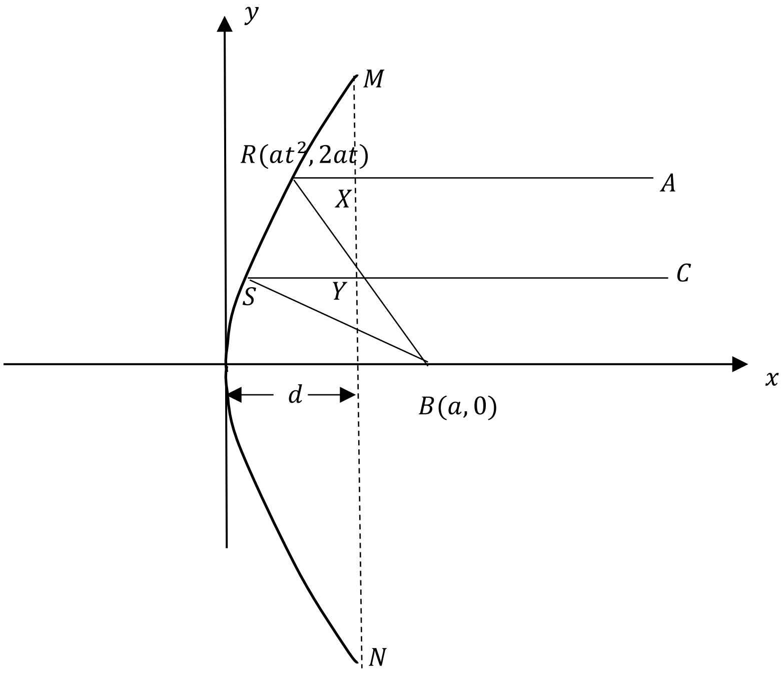

Fig.2 : Total phase difference between two waves due to reflection

Let the height of the reflector be d. Consider two waves AXR and CYS travelling parallel to the main axis of the reflector. These two waves should meet after the reflection at the focal point B (a,0). I f the two waves are emitted at a distant point their phases at X and Y should be the same.

$XR = d - at^2$, $RB = \sqrt{(a - at^2) + (2at)^2}$

Therefore $XR + RB = d - at^2 + \sqrt{(a - at^2) + (2at)^2}$

$$= d - at^2 + \sqrt{a^2 - 2a^2t^2 + a^2t^4 + 4a^2t^2}$$

$$= d - at^2 + \sqrt{a^2 + 2a^2t^2 + a^2t^4}$$

$$= d - at^2 + \sqrt{(a + at^2)^2}$$

$$= d + a$$

This shows that the total distance travelled by a wave from the plane MN to the focal point is a constant, provided that its direction of propagation is parallel to the x-axis.

ie. The two waves are collected 100% in-phase at B.

2.3 The angular incidence of waves on a parabolic reflector

Suppose a wave is incident at an angle $\boldsymbol{\alpha}$ on the reflecto*r.*

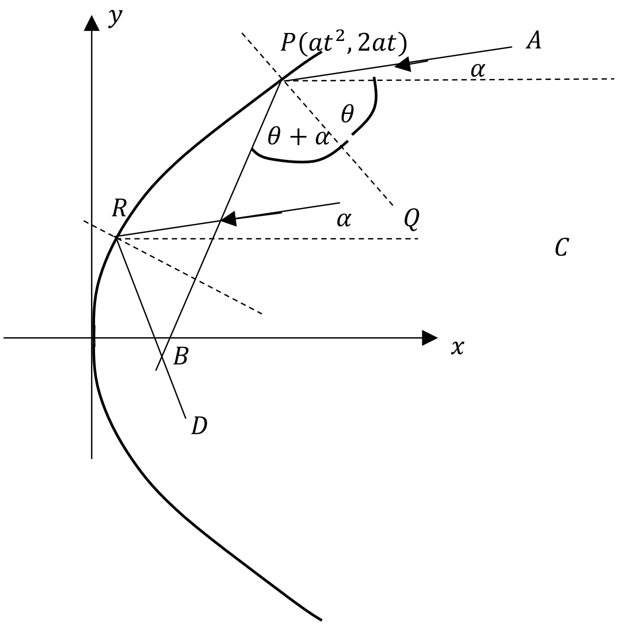

Fig.3: Angular incidence of two waves

Let us consider a wave AP incident on the reflector at an angle $\boldsymbol{\alpha}$ with the x-axis. Since the gradient of the normal PQ *at* P *is* $-t$, $tan(\pi - \theta) = -\boldsymbol{t} \;\Rightarrow\; tan\,\theta = \boldsymbol{t}$

Therefore, the angle of reflection of the wave is $\theta + \alpha$, which is the angle between PB and the normal PQ.

Thus, the angle made by the reflected wave with the $\boldsymbol{x}$-axis is $\pi - (2\theta + \alpha)$

And the gradient of $PB = tan\big(\pi - (2\theta + \alpha)\big) = -\,tan(2\theta + \alpha) = -\frac{tan2\theta + tan\alpha}{1 - tan2\theta.tan\alpha}$

Substituting $tan2\theta = \frac{2tan\theta}{1 - tan^2\theta} = \frac{\mathbf{2t}}{\mathbf{1 - t^2}}$ in the above equation,

The gradient of the reflected wave $PB \; = -\frac{\mathbf{2t + (1 - t^2)tan\alpha}}{\mathbf{1 - t^2 - 2t.tan\alpha}} = m$

Therefore, the equation of PB is $\boldsymbol{y} = m(x - at^2) + 2at$

If any two points $\boldsymbol{S}(at_1^2, 2at_1)$ and $T(at_2^2, 2at_2)$ on the parabola are selected, we write their equations as follows*:*

$y = m_1(x - at_1^2) + 2at_1$ and $\boldsymbol{y} = m_2(x - at_2^2) + 2at_2$ where $m_1 = -\frac{2t_1 + (1 - t_1^2)tan\alpha}{1 - t_1^2 - 2t_1.tan\alpha}$ and

$m_2 = \frac{2t_2 + (1 - t_2^2)tan\alpha}{1 - t_2^2 - 2t_2.tan\alpha}$.

If the two lines intersect at a point $H(x_H, y_H)$ then $x_H = \frac{a(m_1t_1^2-m_2t_2^2)}{m_1-m_2} + \frac{2a(t_2-t_1)}{m_1-m_2}$ and

$$y_H = m_1 \left(\frac{a(m_1t_1^2-m_2t_2^2)}{m_1-m_2} + \frac{2a(t_2-t_1)}{m_1-m_2} - at_1^2\right) + 2at_1$$

This shows that the point of intersection H is not a fixed value but depends on the points of incident of two wave on the parabolic reflector.

3 METHODOLOGY

The design parameters of parabolic reflector to meet the required gain, required side lobe level etc. are discussed by some authors (Jeom-Hun Lee, Seong-Pal Lee et al., 2005) in the past. Shung-Wu Lee et al. (1975) in his paper has discussed amplitude- and phase matching of axial waves using vector field analysis for a smooth conducting reflector. The paper does not discuss about off-axis incidence of waves.

In this work main emphasis is given to off – axial incidence of waves on a reflector. The metallic surface of the reflector was assumed to be smooth and the reflection coefficient is assumed to be -1. The focusing property of the reflector was investigated for various angles of incidence.

Simulation was done using MATLAB. For the simulation work a paraboloid with

$a = 24$ units was used. The highest point of incidence of a wave on the paraboloid was decided from the gradient of the tangent at that point.

Since the gradient of the tangent at a point $P(at^2, 2at)$ is $\frac{1}{t}$, maximum value of $\frac{1}{t}$ was assigned for $\tan\alpha$, where α is the angle made by the incident wave with the x-axis.

i.e. $\frac{1}{t} > \tan\alpha \Rightarrow t < \frac{1}{\tan\alpha}$

Finally, the results of the paraboloid was compared with a spherical reflector.

If (R, φ) is the polar coordinate of the point of reflection of the wave on the spherical reflector - reference point being the center of the sphere, then the gradient of the tangent at the point of reflection is $-\cot\varphi$.

i.e. $-\cot\varphi > \tan\alpha$

For a spherical reflector normally $\varphi > 90^0$. Hence if the acute angle (γ) between the negative x-axis is selected, above equation changes to

$\cot\gamma > \tan\alpha$

4 RESULTS AND DISCUSSION

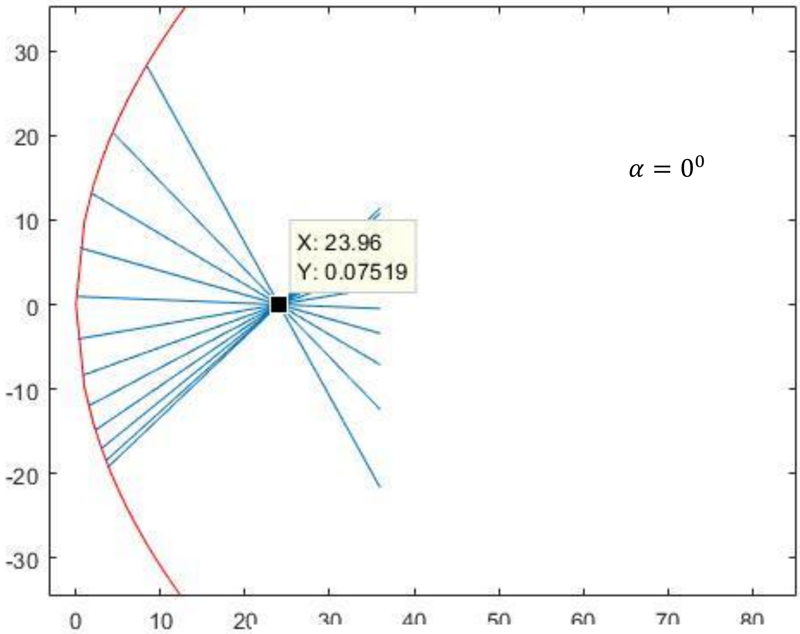

Fig.4: Parabolic reflector with $\alpha = 0^0$

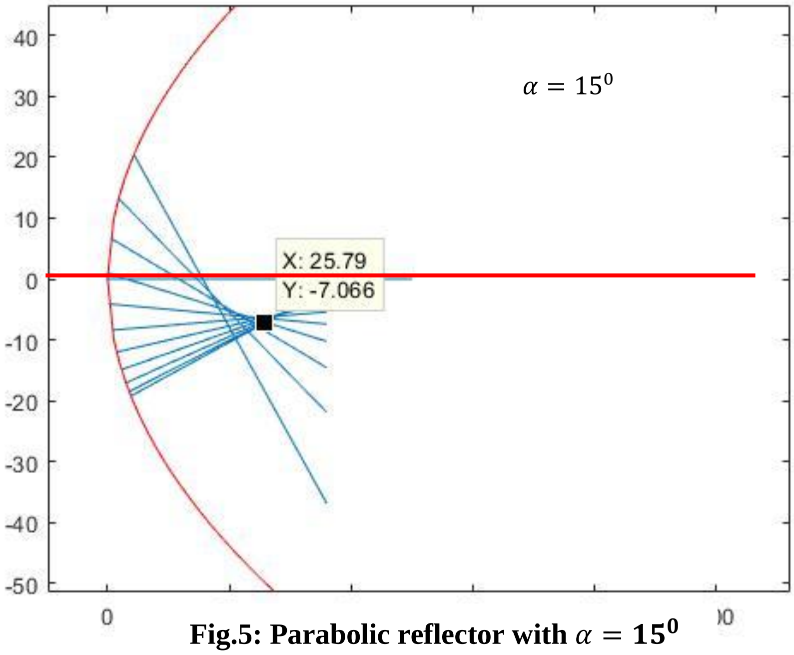

Fig.5: Parabolic reflector with $\alpha = 15^0$

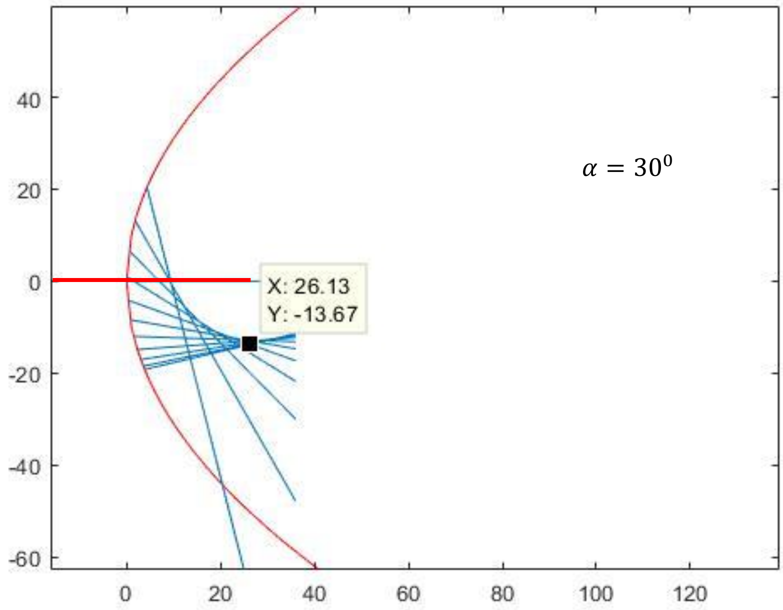

Fig.6: Parabolic reflector with $\alpha = 30^0$

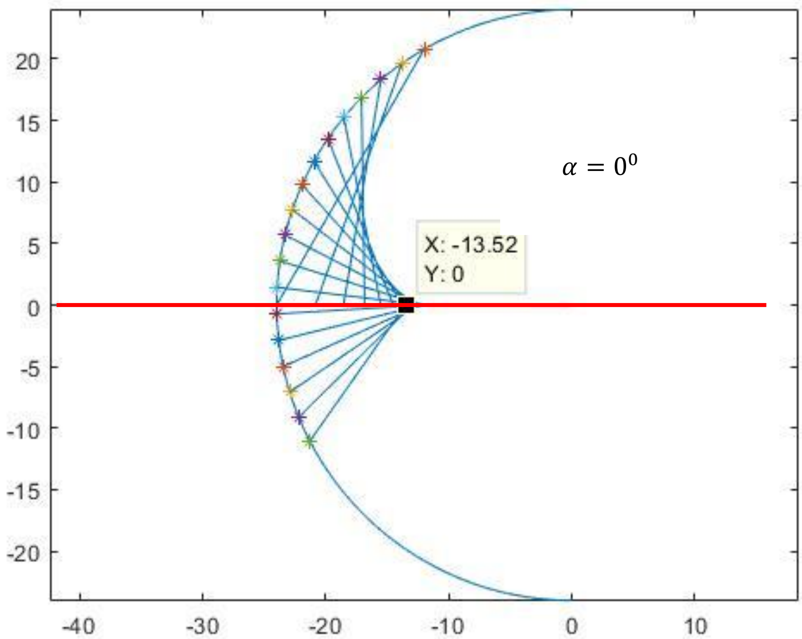

Fig.7: Spherical reflector with $\alpha = 0^0$

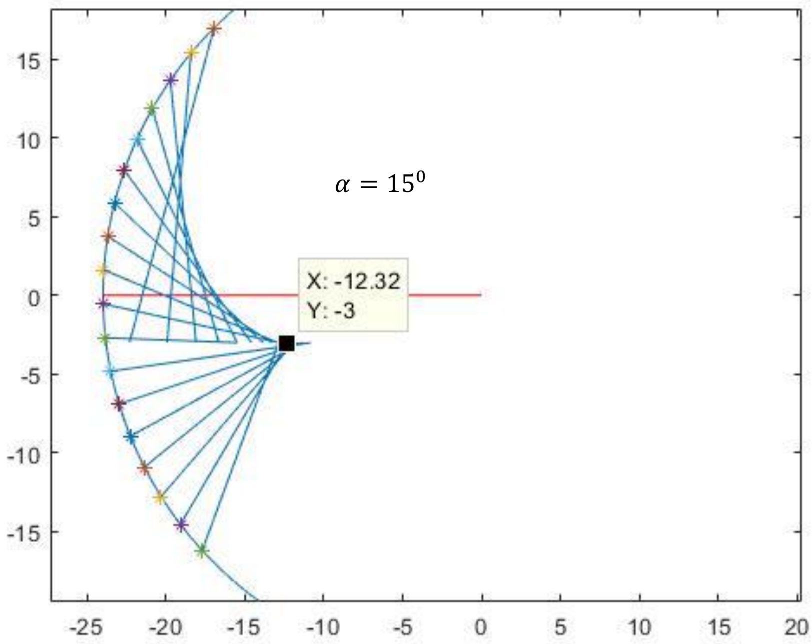

Fig.8: Spherical reflector with $\alpha = 15^0$

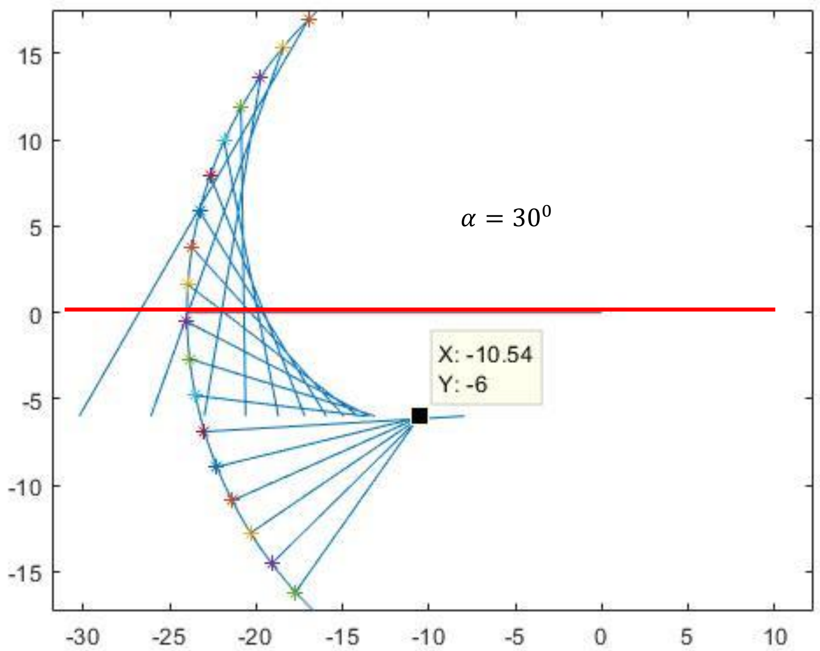

Fig. 9: Spherical reflector with $\alpha = 30^0$

The focusing ability of parabolic- and spherical reflectors for different offset angles are shown above. Only the reflected rays are shown in each figure.

Also investigated is the angular incidence of a radial wave on a spherical reflector (Fig.10). If the spherical reflector is replaced by a parabolic reflector the result is given in Fig. 11.

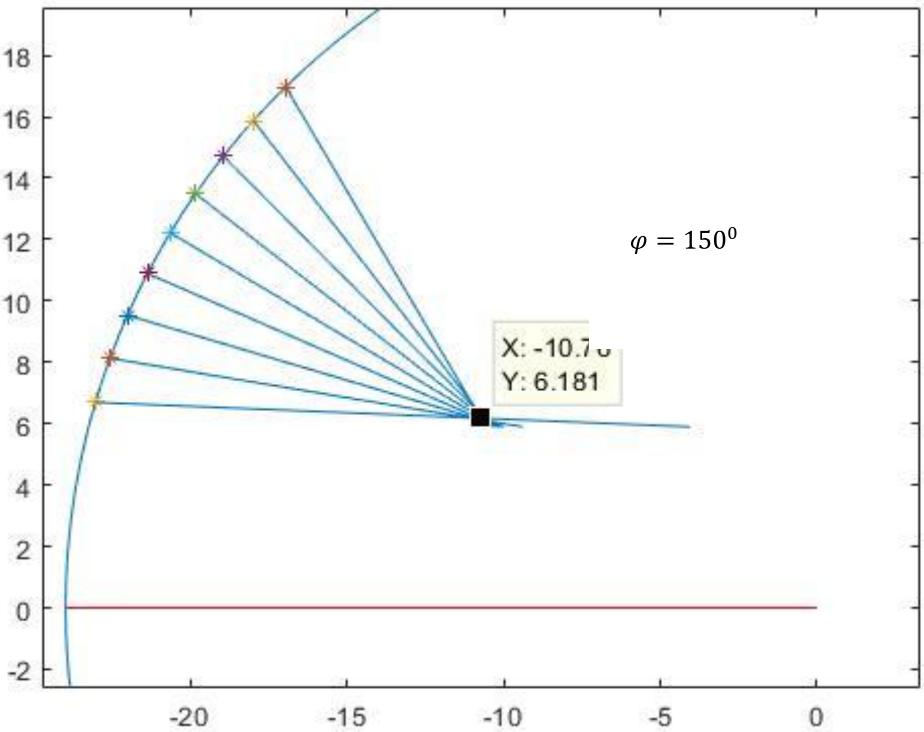

Fig.10: Angular incidence of a radial wave on a spherical reflector at $\varphi = 150^0$

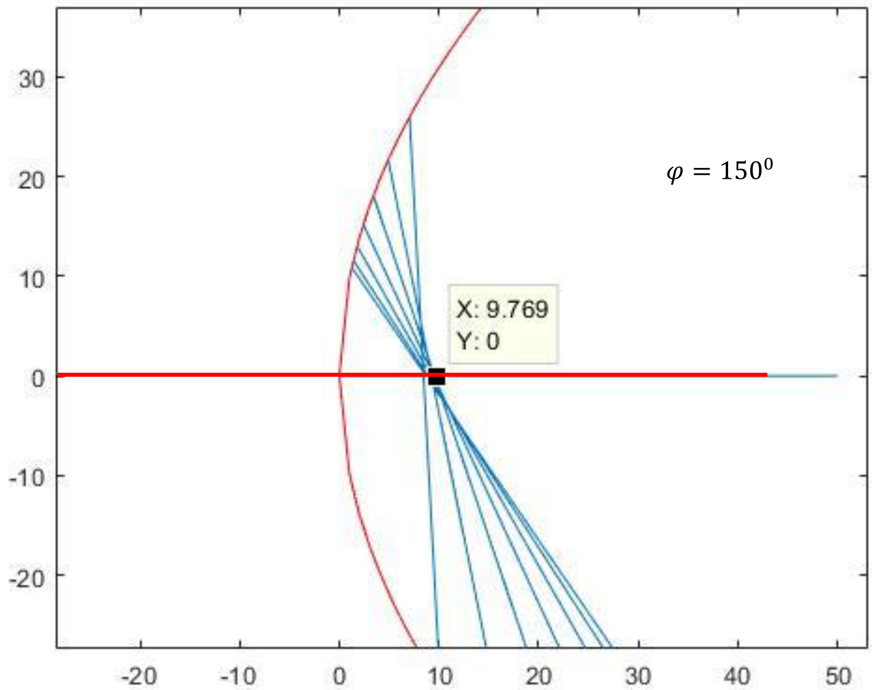

Fig.11: Angular incidence on a parabolic reflector at $\varphi = 150^0$

An ideal reflector antenna for electromagnetic waves should perform following tasks:

(i) The reflector should focus the waves transmitted from a distant transmitter (parallel waves) to a single point.
(ii) The waves collected at the focal point must be in-phase.

Condition (ii) is particularly important for millimeter waves, since the wave length of such a wave is in the order of millimeters. A small change in the phase can reduce the signal strength due to destructive interference.
Fig.4 shows how precisely the paraboloid collects axial waves. All the waves are collected at the focal point located at (24, 0) as predicted in the theory.
Fig.7. shows how the spherical reflector reacts to the waves incident parallel to the x-axis. For a smaller aperture, waves converge to a single point as seen from the figure. For the selected radius of 24 *units* of the sphere, waves are collected at the point (13.5, 0).
When the aperture increases it is obvious that distant waves are not focused properly.

Angular incidence of waves on the parabolic reflector shows that some selected rays incident on a particular region are focused with a reasonable accuracy (Figs. 5 and 6). When the offset angle is increased the focal point tends to shift towards the direction of the negative y-axis.
But the waves beyond this selected region tend to be collected at different locations.
Figs. 8 and 9 show the angular incidence of waves on a spherical reflector. This looks very much similar to the response of the parabolic reflector- The focal point is shifted in the direction of negative y-direction when the angle of incidence is increased.
When the Figs.5, 6, 8 and 9 are carefully investigated, we can see that the waves incident with smaller offset angles tend to converge to a single point.
The focusing ability of reflectors for angular incidence was further investigated for a radial incidence. This is shown in Figs. 10 and 11. The offset angle was set to $\varphi = 150^0$. Both the parabolic reflector and the spherical reflector exhibit good converging properties. For the spherical reflector the focal point has been shifted in the positive y-direction.

5 CONCLUSION

This work was carried out to investigate the angular incidence of e.m. waves on a parabolic reflector. In practice offset incidence on parabolic reflectors are used for domestic TV receivers. As can be seen from results a larger radius of curvature of the reflector can focus more waves, when the offset angle is below 30^0 . In the case of a parabolic reflector larger values of the focal length can exhibit better focusing properties for angular incidence. This can be optimized by making the effective reflector aperture smaller, as can be seen in Fig. 5 and Fig.6. Also the results show that the parabolic reflector and the spherical reflector do not show much difference for angular incidence of waves if larger radii of curvature are used. Also since various waves converging at the focal point (approximate) are not in-phase for offset incidence, the resultant signal strength can be comparatively low due to the destructive interference, for the waves in the SHF and EHF bands. For VHF and UHF bands smaller changes in the phase of the waves cannot influence the total signal strength very much since the wave lengths of the waves comparatively high.

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