CAPTURING LEVERAGE EFFECT OF HIGH VOLATILITY DATA SERIES

T. M. J. A. Cooray[*]

*Department of Mathematics and Philosophy Faculty of Engineering Technology
The Open University of Sri Lanka*

INTRODUCTION

The ARCH model (Autoregressive Conditional Heterocesdasticity) was originally introduced by Engle (1982) as a convenient way of modelling time-dependent conditional variance. This model was generalized by Bollerslev (1986) as the GARCH model (Generalized Autoregressive Conditional Heterocesdasticity). Chiu, H.C. andWang, D. (2006) have shown that the use of these models tends to produce bias in predictions of volatility. On the other hand, authors such as Bollerslev and Wooldridge (1992) have shown the estimation problems of such models, problems that would explain the obtaining of bias in predictions of volatility.

However, the possibility that the conditional variance of these processes (h_t) may not be a valid measure of volatility in time series with time-varying variances has never been considered. In this work, we show that the conditional variance h_t obtained by a GARCH(1,1) model is a biased measure of volatility prediction.

METHODOLOGY

Symmetric GARCH models
The GARCH(1,1) model can be written as

$$y_{t+1} = \mu_{t+1} + e_{t+1}$$

$$e_{t+1} = \sigma_{t+1} u_{t+1}$$

$$u_{t+1|t} \sim N(0,1)$$

$$\sigma^2_{t+1} = \omega + \beta_1 \sigma^2_t + \alpha_1 e^2_t$$

With unconditional variance $\bar{\sigma}^2 = \dfrac{\omega}{1 - \alpha_1 - \beta_1}$ and covariance stationary condition of GARCH(1,1) stationary condition is $1 \geq \alpha_1 + \beta_1$

Integrated GARCH models
Most financial market have GARCH volatility format that mean revert. In other words the volatility converges to an average level in long term. However currencies and commodities tend to have volatilities that are not as mean reverting as volatilities of other types of financial assets. The volatilities of exchange rate may be random walk in such cases usual stationary GARCH models will not apply . Suppose that In addition to that $\alpha_1 + \beta_1 = 1$ and $\alpha = 1 - \lambda$ The GARCH (1,1) model can be written as

$$\sigma^2_{t+1} = \omega + (1-\lambda)e^2_t + \lambda\sigma^2_t \text{ and } \bar{\sigma}^2 = \frac{\omega}{1-(1-\lambda)-\lambda} = \frac{\omega}{0} \rightarrow \infty$$

---

[*] Email tmcor@ou.ac.lk

In this case process is not stationary. Then the equation is called (IGARCH) model when $\omega = 0$ IGARCG model becomes Exponentially weighted moving average (EWMA) model.

Asymmetry Model

In equity market it is commonly observe that volatility is higher in a falling market than it is rising market. Black (1976) was perhaps the first to observe that stock returns are negatively correlated with changes in volatility: that is, volatility tends to rise (or rise more) following bad news (a negative return) and fall (or rise less) following good news (a positive return). This is called the *leverage effect,* as firms use of leverage can provide an explanation for this correlation, if a firm uses both debt and equity then as the stock price of the firm falls its debt-to-equity ratio rises. This will raise equity return volatility if the firm's cash flows are constant. The leverage effect has since been shown to provide only a partial explanation to observed correlation, but the name persists. Given the above correlation we might expect that negative returns today lead to higher volatility tomorrow than do positive returns. This behaviour cannot be captured by a standard GARCH(1,1) model: $\sigma_{t+1}^2 = \omega + \beta_1\sigma_t^2 + \alpha_1 e_t^2$

which shows that tomorrow's volatility is quadratic in today's return, so the sign of today's return does not matter. The simplest extension to accommodate this relation is the model of Glosten, Jagannathan and Runkle (1993) (so-called *GJR-GARCH,* sometimes known as Threshold-GARCH):

$$GJR\text{-}GARCH: \quad \sigma_{t+1}^2 = \omega + \beta_1\sigma_t^2 + \alpha_1 e_t^2 + \delta e_t^2 I\{e_t < 0\}$$

If $\delta > 0$ then the impact on tomorrow's volatility of today's return is greater if today's return is negative. One way of illustrating the difference between a volatility models is via their News impact curves., see Engle and Ng (1993). This curve plots t+1 as the value of t varies, leaving everything else in the model fixed, and normalising the function to equal zero when $e_t = 0$. This is illustrated in Figure 1 below. The news impact curve for a standard GARCH model is simply the function $\alpha_1 e_t^2$ while for the GJR-GARCH model it is $\alpha_1 e_t^2 + \delta e_t^2 I\{e_t < 0\}$

If the two models being compared are nested, in the sense that for certain choices of parameters the models are identical, then we can conduct statistical tests to see if the models are significantly different. The simplest example of this is comparing a GJR-GARCH model with a GARCH model:

GJR-GARCH(1,1) : $\sigma_{t+1}^2 = \omega + \beta_1\sigma_t^2 + \alpha_1 e_t^2 + \delta e_t^2 I\{e_t < 0\}$

GARCH(1,1): $\sigma_{t+1}^2 = \omega + \beta_1\sigma_t^2 + \alpha_1 e_t^2$

When $\delta = 0$ the GJR-GARCH is simply the GARCH model. Taylor's formula, we can compute the standard errors for the GJR-GARCH parameters and we can test:

$H_o : \delta = 0$

$Vs \ H_1 : \delta \neq 0$

If we reject the null hypothesis then we conclude that the GJR-GARCH(1,1) is significantly better than the GARCH(1,1) model, at least in-sample.

RESULTS AND DISCUSSION

The results from estimating a *GJR-GARCH*(1,1) on the three series analysed above are presented below. Parameters that are significantly different from zero at the 5% level are denoted with an asterisk. From the Table 1, we see that the asymmetry parameter, δ is significant for the stock return and the interest rate, but not for the exchange rate. Notice that the significant δ estimates are both positive, indicating that negative shocks lead to higher future volatility than do positive shocks of the same magnitude.

GJR-GARCH (1,1) parameter estimates				
parameter	ω	α_1	β_1	δ
DM/USD exchange rate	0:0054*	0:0397*	0:9490*	-0:0014
FTSE 100 index	0:0015*	0:0209*	0:9496*	0:0389*
US 3-month T-bill rate	0:0233*	0:0940*	0:7498*	0:1535*

Table 1 Parameter estimates of *GJR-GARCH* Model for different series

By modelling σ^2_{t+1} , we are ensured a positive estimate of σ^2_{t+1} . Further by allowing to differ from zero the leverage effect can be captured. The parameter estimates for *GJR-GARCH*(1,1) models estimated on our three data sets are presented in the Table 1. Parameters that are significantly different from zero are denoted with an asterisk , the *GJR-GARCH* model results indicate that the asymmetry parameter, in this case, is significant for the stock return and the interest rate but not for the exchange rate. Here notice that all estimates are negative, which again implies that negative shocks lead to higher future volatility than do positive shocks of the same magnitude.

CONCLUSIONS

In this paper we examine two models for asymmetries, namely dynamic leverage and threshold effects, in Stochastic Volatility (SV) models, one based on the threshold effects and the other on dynamic leverage (DL), or the negative correlation between the innovations in returns and volatility. Three financial time series are used to estimate the SV models, with empirical asymmetric effects found to be statistically significant in each case. The empirical results for FTSE 100 index, US 3-month T-bill rate indicate that dynamic leverage dominates the threshold effects model for capturing asymmetric behavior, while the results for DM/USD exchange rate show that dynamic leverage and threshold effects models are rejected against each other.
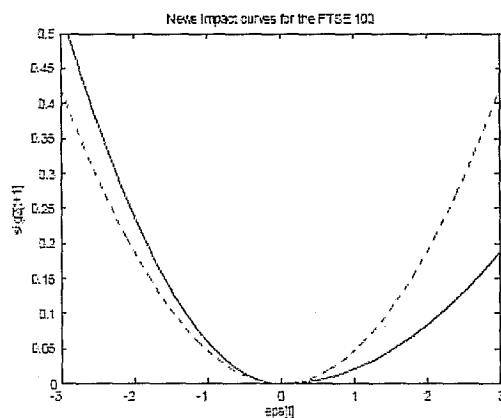

Figure 1: The news impact curves of a *GJR-GARCH(1,1)* and a standard GARCH(1,1) model.

REFERENCES

Black, F. 1976. "Studies in Stock Prices Volatility Changes." *Proceedings from the American Association, Business and Economic Statistic Section.* 177-81.

Bollerslev, T. (1986). Generalised Autoregressive Conditional Heteroscedasticity. Journal of Econometrics, 31, 307-27.

Bollerslev, T. and Wooldridge, J. (1992). Quasi-maximum likelihood estimation and inference in dynamic models with time varying covariances. Econometric Reviews,11, 143-72.

Brooks C. (2004). Introductory Econometrics for Finance. Cambridge University Press.

Chiu, H.C. andWang, D. (2006). Using conditional autoregressive range model to forecast volatility of the stock indices. *Proceedings of Joint Conference on Information Science 2006.* Atlantis Press.

Engle, R. (2001). The use of ARCH/GARCH Models in Applied Econometrics. *Journal of Economic Perspectives* 15: 157-168

Engle, R. F. 1982. "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation." *Econometrica.* 50(4). 987-1008.6.

Engle, R.F. and V.K. Ng (1993). Measuring and testing the impact of news on volatility. *Journal of Finance*, 48(5), 1749-78.

Engle, R.F. (1995). *ARCH: Selected Readings.* New York: Oxford University Press.

Glosten, L., R. Jagannathan, and D. Runkle (1993): "On the Relation between Expected Return on Stocks," Journal of Finance, 48, 1779-1801.

Henry, O., (1998) "Modelling the asymmetry of stock market volatility", Applied Financial Economics (8), pp 145-153

Koutmos, G. and R. Saidi, (1995) "The leverage effect in individual stocks and the debt to equity ratio", Journal of Business Finance and Accounting (22), pp 1063-1073

Taylor, S. J. (1986), Modelling Financial Time Series Wiley, Chischester.