Optimizing the Cost of the Delivering Process of a Production Company: A Mathematical Model

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Abstract – *In this study we develop a mathematical model to optimize the transportation cost of a food company to deliver their product consisting of 45 numbers of items which are delivered to be several customer points on their demand using 6 numbers of vehicles. Using cost function as the object function, demand at each destination and supply by a vehicle; we identify the optimal way to deliver the production items by reducing relevant costs.*

Keywords*: labour cost per Km (R_L), fuel cost per Km (R_F), vehicle maintenance cost per Km (R_M).*

1 INTRODUCTION

The transportation model is a special class of linear programming that deal with shipping a commodity from sources (e.g., factories) to destinations (e.g., warehouses)[2]. The objective is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits. The model assumes that the shipping cost is proportional to the number of units shipped on a given route. In general, the transportation model can be extended to other areas of operation, including, among others, inventory control, employment scheduling, and personnel assignment [2].

A leading food production company needs to minimize the total transportation cost with available vehicles in order to increase its production and expand its national markets. The company launched their business over 70 years ago. They have grown to be the one of the unquestioned leaders in the field and the company has become a household word for good reliable food. They produce variety of household items such as, food like rice, kurakkan and chillies,etc, covering more than 45 items and deliver over the country covering the market with a wide range of items, throughout the length and breadth of the island.

In this study, we consider their factory located at a particular region as they distribute items keeping the main center at the region. Thus by developing a model it can be mapped to other region where the company has its warehouses.

The transportation cost includes mainly labour cost, vehicle maintenance cost and fuel cost. The labour work is to load the products at the reload points and unload products at customer sites. The fuel cost differs from capacity of the vehicle. The company producing 45 different items and they deliver the items based on the demand at each customer sites by its vehicles. This company has 6 vehicles with similar capacity. It is known that the usage of fuel increases as the distance travelled but also the size of the load.

In this study, we propos a mathematical model to optimize the transportation cost of the food company to deliver their products consisting of 45 numbers of items which are to be delivered to several customer points on their demands using 6 numbers of vehicles.
As our main objective is to minimize the transportation cost, we introduce a cost function which depends on several variables such as vehicle capacity, volume of packs of each item, the distance to the delivering points from loading point, number of packs required at each customer site, labor cost related to transport, maintenance cost of vehicle and etc. The unit cost of transportation per pack is calculated based on the distance travel by a vehicle and adjusted rate is defined on fuel cost, labour cost, and maintenance cost per Km (R_M) we assume that maximum number of packs are loaded to a vehicle.
Using this cost function as the object function, demand at each destination and supply by a vehicle we identify the optimal objective way to deliver the production items by reducing relevant costs.
In this process, it is of partial significant to organize the whole delivery process optimizing the cost of delivery in which the demand of each customer point should be covered on time. The demand can be obtained regularly on the basis daily, weekly or monthly scheduled according to the reports of agents.
A mathematical model is needed to optimize the delivering process. Some of the relevant factors are fuel cost, distance to the delivery points, demand, volume of packs of each item, capacity of the vehicles, labour cost, maintenance cost, capacity of warehouses, limitations of the capacity of producing food items and etc.

2 METHODOLOGY

In generally, transportation model is an iterative procedure for solving problems that involves minimizing the cost of transportation from a series of sources to a series of destinations. However in this study we focused on minimizing the cost of transportation using limited vehicles of a company and therefore, we assume that the sources are represented the items (in packet form) loaded to vehicles. The transportation cost per packet is calculated based on the cost of fuel, cost of labour and cost of maintenance with respect to the distance traveled by the assigned vehicles.

To formulate the model, the following information were used:

- Unit cost per packet
- Distance matrix from the origin to destination
- The total number of items loaded to a vehicle

Transportation problems give rise to the simplest kind of linear program for minimizing cost flows. We then generalize to a transportation model, an essential step if we are to manage all the data, variables and constrains effectively.

1. The origin points and the capacity or supply per period at each.
2. The destination points and the demand per period at each.
3. The cost of delivering one unit from each origin to each destination.

2. 1 Mathematical Model

The transportation cost can be obtained using the following objective function:

$$F = Min \sum_{i=1}^{N_v} \sum_{j=1}^{N_p} C_{ij} P_{ij} \quad (1)$$

where,

P_{ij} is the number of packets from i^{th} source (vehicle) to j^{th} destination point.

N_P is the total number of packets loaded to vehicles and

N_V is total number of vehicles available.

C_{ij} is the unit cost for transforming a packet from i^{th} source (vehicle) to j^{th} destination point and it is defined by,

$$C_{ij} = \alpha \times d_{ij}, \quad (2)$$

where, d_{ij} is the distance from the i^{th} vehicle to j^{th} destination point and

α is the adjustable cost per packet and is defined by

$$\alpha = \frac{R_a}{N_p} \quad (3)$$

Where is is the adjustable rate:

$$Ra = R_F + R_L + R_M \quad (4)$$

[The supply and demand as well as the unit cost of each packet transported by each vehicle to different destination (say there is N destinations)]. The total supply(S) is sum of the items loaded to a vehicle as required at each destination and demand (D) is the total amount of items loaded to a each vehicle are shown in table 1:[1,5]

Table 1 Total number of items loaded to vehicles

	1	2	3	…	N	S
V_1	P_{11}	P_{12}	P_{13}	..	P_{1N}	$\sum_{j=1}^{N_{p3}} p_{1j}$
V_2	P_{21}	P_{22}	P_{23}		P_{2N}	$\sum_{j=1}^{N_{p3}} p_{2j}$
V_3	P_{31}	P_{32}	P_{33}	..	P_{3N}	$\sum_{j=1}^{N_{p3}} p_{3j}$
..	..	..	..	..	..	..
D	$\sum_{i=1}^{N_v} p_{i1}$	$\sum_{i=1}^{N_v} p_{i2}$	$\sum_{i=1}^{N_v} p_{i3}$	..	$\sum_{i=1}^{N_v} p_{im}$	$\sum_{i=1}^{N_v}\sum_{j=1}^{N_p} p_{ij}$

The cost matrix of each entry, Cij is the unit cost per packet for transforming from i^{th} source (vehicle) to j^{th} destination point. It can be calculated using the above formula [3].

Table 2 Unit Cost for transforming packets from i^{th} source to j^{th} destination point

	1	2	3	…	**N**
V_1	C_{11}	C_{12}	C_{13}	…	C_{1N}
V_2	C_{21}	C_{22}	C_{23}	…	C_{2N}
V_3	C_{31}	C_{32}	C_{33}	…	C_{3N}
…	…	…	..	…	..
V_{N_v}	C_{Nv1}	C_{Nv2}	C_{Nv3}	…	C_{NvNp}

3 RESULTS AND DISCUSSION

Numerical Example:

The proposed model is used to find the optimal transshipment of items by 6 vehicles to 10 destinations. The demand at each destination point is given as 10, 30, 40, 0, 30, 40, 25, 30, 55, and 40 and also the supply by each vehicle are taken as 50 packets.

The distance between the source where vehicle is parked to the destination point is given in the table 3.

Assume that capacity of vehicles are the same and suppose that R_F= 27.14, R_L= 4.25, and R_M = 2.75 as per the industry standard.

Then using equation (4) we get Ra = $R_F + R_L + R_M$ =34.14. The total number of packets (Np) loaded to all vehicles is 50x6=300.

Hence using equation (3) we get, the adjustable rate is $\alpha = 0.1138$ and hence the unit cost for transform a packet from a loaded point i to customer site j is calculated using the equation (2) and it is tabulated in table (4).

Table 3 Distance matrix

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}
V_1	25	95	20	40	50	90	60	20	45	55
V_2	25	95	20	40	50	90	60	20	45	55
V_3	25	95	20	40	50	90	60	20	45	55
V_4	25	95	20	40	50	90	60	20	45	55
V_5	25	95	20	40	50	90	60	20	45	55
V_6	25	95	20	40	50	90	60	20	45	55

Table 4 Unit cost for transforming a packet by i^{th} vehicle to j^{th} the customer site

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}
V_1	2.84	10.81	2.28	4.55	5.69	10.24	6.83	2.28	5.12	6.26
V_2	2.84	10.81	2.28	4.55	5.69	10.24	6.83	2.28	5.12	6.26
V_3	2.84	10.81	2.28	4.55	5.69	10.24	6.83	2.28	5.12	6.26
V_4	2.84	10.81	2.28	4.55	5.69	10.24	6.83	2.28	5.12	6.26
V_5	2.84	10.81	2.28	4.55	5.69	10.24	6.83	2.28	5.12	6.26
V_6	2.84	10.81	2.28	4.55	5.69	10.24	6.83	2.28	5.12	6.26

Based on the above inputs we obtain the final the optimal solution by least cost method as given below:

Table 5 Optimal Model allocation for the sample data

Vehicles	DP_1	DP_2	DP_3	DP_4	DP_5	DP6	DP_7	DP_8	DP_9	DP_{10}	Supply
V_1			40					10			50
V_2	5			0				20	25		50
V_3					20				30		50
V_4					10					40	50
V_5		25					25				50
V_6	5	5				40					50
Demand	10	30	40	0	30	40	25	30	55	40	300

The results show that the vehicle 1 is assigned to transship the 40 packets to destination point 3(DP3) and 10 packets to the destination point 8 and mean time vehicle 2 is used for transship the 5 packets to destination point 1 and 20 packets to destination point 8 and 25 packets to the point 9, and similarly the other vehicles are used as shown in the table 5. The total cost for the transshipment using the 6 vehicles. Total transportation cost is

$$F=(2.28)(40)(20)+(2.28)(10)(20)+(2.84)(5)(25)+(2.28)(20)(20)+(5.12)(25)(45)+(5.69)(20)(50)+(5.12)(30)(45)+(5.69)(10)(50)+(6.26)(40)(55)+(10.81)(25)(95)+(6.83)(25)(60)+(2.84)(5)(25)+(10.81)(5)(95)+(10.24)(40)(90)$$

$$=1824+456+355+912+5760+5690+6912+2845+13772+25673.75+10245+355+5134.75+36864$$

$$= 116798.50$$

4 CONCLUSIONS / RECOMMENDATIONS

The proposed transportation model can be used for Production Company for finding optimal transshipment of their products. In particularly, this model can be used find the maximum usage of vehicles by reducing the transportation cost. This model introduces a flexible formula for computing rates related to distance traveled to delivery points. All the important factors related to transportation cost are considered in calculating unit cost matrix. They can distribute the packs of each item, so the number of items included in the pack can be decided on the demand on the particular item and capacity of vehicle. The proposed model can be automated whenever the company decides new adjustable rate or any other factors such as rate of deterioration of vehicle parts, road with slops and bends, etc.

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