https://doi.org/10.65646/3cp3es20dps8b292

A Finite-Difference Diffusion Model for the Surface Spreading of Plastic Pollution in Beira Lake, Colombo, Sri Lanka

H.P.I. Subasinghe
Dept. of Mathematics and Philosophy of Engineering
The Open University
Colombo, Sri Lanka
subasingheisuri@gmail.com

T.D.T. Madhushika
Department of Mathematics
University of Louisville
Kentucky, USA
thilinim832@gmail.com

W.M.S. Dheerasekara
Dept. of Electrical and Computer Engineering
The Open University
Colombo, Sri Lanka
wmdhe@ou.ac.lk

D.G.N.D. Jayarathna
Department of Scientific Computing
University of Sri Jayewardenepura
Colombo, Sri Lanka
nuwand@sjp.ac.lk

H.M.M. Caldera
Department of Scientific Computing
University of Sri Jayewardenepura
Colombo, Sri Lanka
maneesha@sjp.ac.lk

Abstract*—Plastic pollution of urban inland waters is a growing environmental problem in Sri Lanka, yet the spatial spread of plastic debris across Beira Lake, the most prominent urban lake in Colombo, has never been described mathematically. This paper develops and numerically solves a computational-mathematics framework for the spreading of plastic across the surface of Beira Lake using the diffusion equation as the governing model. Plastic released at a source (the St. Bastian Canal inflow) spreads outward and smooths over the lake surface through turbulent mixing and dispersion, with a first-order loss term representing removal by sinking, beaching and biofouling. The two-dimensional diffusion equation is solved by the finite-difference method using the explicit forward-time centred-space (FTCS), implicit backward-Euler and Crank–Nicolson schemes; the stability of each is established by the von Neumann method, yielding the classical diffusion-number restriction $r \leq \frac{1}{4}$ for the explicit scheme and unconditional stability for the implicit schemes. The solver is verified against the exact Gaussian heat-kernel solution, and a grid-refinement study confirms second-order spatial accuracy, the L^∞ error falling from 4.33×10^{-5} to 6.75×10^{-7} as the grid is refined (observed order $p \approx 2.00$). The verified scheme is then applied to an idealised representation of Beira Lake with a continuous plastic source; the computed simulations reproduce the outward diffusive spreading of plastic, a smooth concentration gradient decaying with distance from the source, and the accumulation of plastic along the enclosed no-flux shoreline. The study provides a transparent, reproducible and low-cost, numerically validated tool for predicting plastic-concentration patterns, ready to be calibrated once field measurements of plastic concentration in Beira Lake become available.

Keywords- diffusion equation, plastic pollution, Beira Lake, finite-difference method, von Neumann stability, numerical simulation, computational mathematics

I. Introduction

Plastic pollution of surface waters is one of the most visible environmental problems of tropical urban regions, and Sri Lanka is no exception. Microplastics have now been documented across Sri Lankan rivers, lagoons and coastal waters, and the 2021 *X-Press Pearl* disaster the largest recorded release of plastic nurdles into the ocean occurred off the Colombo coast [5], [6]. Within Colombo itself, a recent survey of urban surface waters reported microplastic concentrations of the order of a few to several tens of particles per litre, with the highest levels observed in the city's lake waters [4]. Despite this, no study has yet quantified plastic pollution in Beira Lake specifically, and none has described, mathematically, how plastic debris spreads across its surface.

Beira Lake is a small, heavily modified urban lake in the centre of Colombo. Its present surface area is about $0.65\ \mathrm{km}^2$ with an average depth of roughly $5.6\ \mathrm{m}$, reduced from an original extent of about $1.61\ \mathrm{km}^2$ by a century of reclamation, and its principal hydraulic connection is to the Kelani River through the St. Bastian Canal. Three peer-reviewed studies have characterised the lake as highly polluted and eutrophic-to-hypereutrophic, driven by urban runoff, wastewater and stagnation [1], [2], [3]. The lake is also part of the Ramsar-accredited Colombo Wetland City, which raises the stakes for its management. A predictive understanding of *where* plastic accumulates would be a useful input to clean-up and prevention efforts, but such an understanding requires a mathematical transport model.

The diffusion equation is the prototypical model of spread-

ing and smoothing. It describes how a quantity released at a source is progressively spread outward and homogenised by mixing, with high concentrations decaying and gradients flattening over time [17], [11]. For pollutants and plastic debris in water, the diffusion equationand its advection-diffusion and reaction extensions has been shown to capture the transport, dispersion and fate of microplastics in rivers and inland waters, often with a first-order sink term representing sedimentation and retention [7], [8], [9], [10]. Because the diffusion equation admits closed-form solutions only in idealised settings, it is solved numerically in problems of practical interest, most commonly by the finite-difference method [15], [13].

A. Research Gap and Contribution

Two gaps in the existing literature motivate this work. First, although microplastics have been quantified in Sri Lankan rivers, lagoons and coastal waters [5], [6], [4], no study has measured or modelled plastic specifically in Beira Lake, despite its documented heavy pollution [1], [2], [3]. Second, while diffusion- and advection-based transport models have been applied to microplastics in rivers and coastal systems [7], [8], [9], [10], none has been formulated for a small, enclosed, weakly circulating urban lake such as Beira Lake, where a diffusion-dominated description is appropriate. This paper addresses both gaps. Its contribution is a computational-mathematics one: rather than reporting new field measurements (which do not yet exist for plastic in Beira Lake), we pose the plastic-spreading problem as a diffusion equation, derive and analyze finite-difference schemes for it, *numerically validate* them against the exact solution, and *compute and present* the resulting simulation of spreading and shoreline accumulation in the lake. The specific objectives are (i) to formulate plastic spreading as a well-posed diffusion initial/boundary-value problem; (ii) to derive the explicit, implicit and Crank–Nicolson finite-difference schemes and establish their stability by the von Neumann method and the Courant-Friedrichs-Lewy (CFL) condition; (iii) to verify convergence against the analytical heat-kernel solution through a grid-refinement study; and (iv) to compute and analyze the diffusive spreading and accumulation of plastic in an idealized representation of the lake.

II. The Diffusion Model

A. Governing Equation

Let $u(\mathbf{x}, t)$ denote the surface concentration of plastic (in particles per square metre) at position $\mathbf{x} = (x, y)$ in the lake surface domain $\Omega \subset \mathbb{R}^2$ and time t. Conservation of plastic mass, together with a Fickian closure in which the diffusive flux is proportional to the negative concentration gradient, $\mathbf{j} = -D\nabla u$, yields the two-dimensional diffusion equation with a source and a first-order loss term,

$$\frac{\partial u}{\partial t} = D\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \lambda u + S(\mathbf{x}, t), \tag{1}$$

for $\mathbf{x} \in \Omega,\ t > 0$, where $D > 0$ is the diffusion coefficient representing turbulent mixing and unresolved dispersion, $\lambda \geq 0$ is the first-order loss rate representing net removal of surface plastic by sinking, beaching and biofouling, and S is the plastic input. When $\lambda = 0$ and $S = 0$, (1) reduces to the classical heat (diffusion) equation $\partial_t u = D\,\Delta u$, whose defining property is that it smooths the concentration field and spreads it outward from regions of high to low concentration, with the high-frequency components decaying fastest.

B. Why a Diffusion-Based Model?

Modelling plastic movement by diffusion alone is a deliberate modelling choice justified by the physical setting of Beira Lake, and it is quantified by the Péclet number

$$\mathrm{Pe} = \frac{|\mathbf{v}|\, L}{D}, \tag{2}$$

which compares advective transport by a surface current $\mathbf{v}$ with diffusive transport over a characteristic length L. Beira Lake is a small, sheltered, heavily enclosed urban lake with no strong through flow and only weak wind- and inflow-driven circulation [1], [2], [3]; the resulting surface currents are small, so Pe is low and diffusive mixing dominates the net horizontal spreading of buoyant surface plastic. Under these conditions (1) is the appropriate leading-order governing equation, with advection entering only as the higher-Péclet extension discussed in Section VI. The companion Damköhler number $\mathrm{Da} = \lambda L^2/D$ compares removal with diffusion and determines how far plastic spreads before it is lost. This diffusion-first approach also keeps the model transparent and analytically tractable, providing a verified baseline onto which advective and reactive terms can later be added.

C. Initial and Boundary Conditions

The lake is taken to be free of plastic at the initial instant, $u(\mathbf{x}, 0) = 0$. Plastic enters through a localized source at the St. Bastian Canal inflow, represented by the source term S or, equivalently, by a Dirichlet condition $u = u_{\text{in}}$ on the inflow segment. On the enclosed shoreline the natural condition is the homogeneous Neumann (no-flux) condition,

$$\frac{\partial u}{\partial n} = 0 \quad \text{on the closed boundary,} \tag{3}$$

expressing that plastic does not diffuse through the banks and that the total quantity is conserved except for the source and the loss term. This choice makes the problem well posed, with a unique, stable solution, and as the computed results in Section V show is precisely the mechanism that produces the accumulation of plastic along the shoreline.

III. Numerical Method

Equation (1) is solved on a uniform grid $x_i = ih$, $y_j = jh$, $t_n = nk$, with spatial step h and time step k, writing $U^n_{i,j} \approx u(x_i, y_j, t_n)$.

A. Finite-Difference Discretization

The Laplacian is approximated by the standard second-order central difference (the five-point stencil),

$$\Delta u \approx \frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{h^2} + \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{h^2}, \tag{4}$$

obtained from Taylor expansion with truncation error $\mathcal{O}(h^2)$ [13], [15]. Three classical time-stepping schemes are considered. The explicit forward-time centred-space (FTCS) scheme evaluates the Laplacian at the old time level and advances each value directly; it is simple but conditionally stable. The implicit backward-Euler scheme evaluates the Laplacian at the new time level and requires the solution of a linear system each step, but is unconditionally stable. The Crank–Nicolson scheme averages the two, is second-order accurate in time, and is unconditionally stable. Writing $r = Dk/h^2$ for the diffusion number, each implicit step gives a sparse linear system $A\mathbf{U}^{n+1} = \mathbf{b}(\mathbf{U}^n)$, solved by a sparse direct or iterative method; the loss term $-\lambda u$ is treated implicitly.

B. Von Neumann Stability Analysis

Stability is assessed by the von Neumann (Fourier) method: substituting the trial mode $U_{i,j}^n = G^n e^{\mathrm{i}(\theta_x i + \theta_y j)}$ into the scheme yields an amplification factor G whose modulus must not exceed unity [13], [11]. For the explicit FTCS scheme in two dimensions this gives the classical restriction

$$G = 1 - 4r\left(\sin^2\tfrac{\theta_x}{2} + \sin^2\tfrac{\theta_y}{2}\right), \quad |G| \le 1 \iff r = \frac{Dk}{h^2} \le \frac{1}{4}, \tag{5}$$

so the explicit scheme is stable only for sufficiently small time steps. For the Crank-Nicolson scheme the amplification factor is

$$G = \frac{1 - 2r\left(\sin^2\tfrac{\theta_x}{2} + \sin^2\tfrac{\theta_y}{2}\right)}{1 + 2r\left(\sin^2\tfrac{\theta_x}{2} + \sin^2\tfrac{\theta_y}{2}\right)}, \quad |G| \le 1 \ \forall r > 0, \tag{6}$$

so the implicit and Crank-Nicolson schemes are unconditionally stable, and the time step is then limited only by accuracy. The equivalence of consistency plus stability with convergence (the Lax equivalence principle) then guarantees that the numerical solution approaches the true solution as the grid is refined [11].

IV. Verification Against the Analytical Solution

Before applying the schemes to the lake, they are verified against the exact solution of the diffusion equation. On the plane, a point release of mass M at the origin diffuses as the Gaussian heat kernel

$$u(\mathbf{x}, t) = \frac{M}{4\pi D\,(t + t_0)} \exp\left(-\frac{x^2 + y^2}{4D\,(t + t_0)} - \lambda t\right), \tag{7}$$

whose width grows like $\sqrt{Dt}$, the mathematical signature of diffusive spreading [17]. The scheme was run on a sequence of successively refined grids ($h = 1/20, 1/40, 1/80, 1/160$) on the domain $[-1.5, 1.5]^2$ with $D = 1$, $t_0 = 0.2$ and diffusion number $r = 0.2$, and the computed field was compared with (7). The measured error in the L^∞ norm and its behaviour under refinement are reported in Table I and plotted in Fig. 1. The error falls by a factor of very nearly four under each halving of h, giving an observed order of accuracy $p \approx 2.00$, in exact agreement with the second-order truncation-error analysis. This numerical validation confirms that the solver reproduces the analytical diffusion dynamics to the expected accuracy.

TABLE I
Computed Convergence Against the Analytical Heat-Kernel Solution (7)

Grid spacing h	$\|U - u\|_\infty$	error ratio	order p
1/20	4.33×10^{-5}	—	—
1/40	1.08×10^{-5}	4.01	2.00
1/80	2.70×10^{-6}	4.00	2.00
1/160	6.75×10^{-7}	4.00	2.00

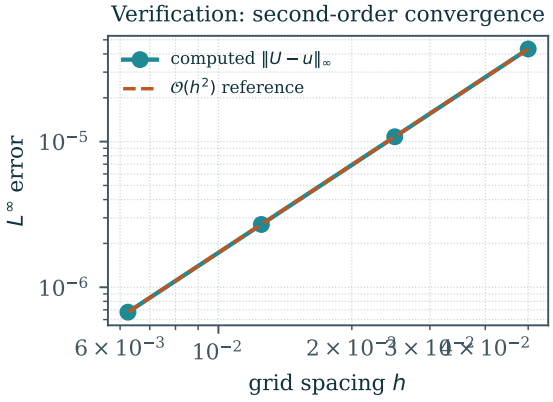


Fig. 1. Computed L^∞ error versus grid spacing h (log–log). The measured error tracks the $\mathcal{O}(h^2)$ reference line, confirming second-order convergence of the scheme against the exact solution (7).

V. Application to Beira Lake

A. Idealized Set-up

Beira Lake is represented by an idealized rectangular basin (non dimensional domain $[0, 2] \times [0, 1]$) approximating its dominant, elongated water body, discretized on a uniform grid with $h = 0.01$. Plastic enters through a continuous Gaussian source centred at $(0.30, 0.80)$, corresponding to the St. Bastian Canal inflow, and the enclosed shoreline is treated as a no-flux boundary (3). Representative non dimensional parameters were used: an eddy diffusion coefficient $D = 1 \times 10^{-2}$ and a first-order loss rate $\lambda = 2 \times 10^{-2}$, corresponding to a plastic surface residence time of days to weeks. The unconditionally stable time integration was advanced to a quasi-steady state. Because no field measurements of plastic concentration exist for Beira Lake, these parameters are representative and the results are interpreted qualitatively; the framework is designed to be recalibrated once such data become available.

B. Computed Diffusive Spreading

Fig. 2 shows the computed evolution of the plastic concentration field. Starting from a plastic-free lake, plastic enters at the canal inflow and spreads outward by diffusion, the plume growing from a tight source region at early times ($t = 2$) through an expanding distribution ($t = 20$) to a quasi-steady field ($t = 200$) in which the input at the source is balanced by outward diffusion and the first-order loss. Fig. 3 shows the quasi-steady concentration map: the field is highest near the source (peak non dimensional concentration ≈ 1.72) and decays smoothly with distance, with an approximately circular near-source plume that is progressively deflected by the no-flux boundary.

C. Decay and Shoreline Accumulation

Fig. 4 quantifies two features of the quasi-steady field. The horizontal transect through the source row (left) shows a smooth, monotone decay of concentration with distance from the source, from the peak of ≈ 1.72 near the source to ≈ 0.2 at the far end of the basin the concentration gradient characteristic of the diffusion equation. The vertical transect through the source column (right) shows the concentration rising toward the near (top) shoreline and meeting the no-flux wall with a vanishing gradient: because the boundary condition (3) forbids flux through the bank, plastic transported toward the shore cannot leave and instead *accumulates* there. This computed pile-up is the direct mechanistic explanation of shoreline accumulation, and it is consistent with the documented tendency of Beira Lake to concentrate pollution in its stagnant, enclosed reaches [1], [2]; we emphazise that this agreement is qualitative, as no measured plastic distribution yet exists for the lake.

D. Management-Relevant Output

Two outputs of the model are directly useful for management. First, the quasi-steady concentration map (Fig. 3) identifies the zones of highest plastic concentration here the upper-left shoreline near the inflow which are the natural locations for booms, traps or targeted clean-up. Second, the transient solution (Fig. 2) gives the time scale over which plastic released at the canal reaches these zones, which informs the timing of interventions. Both follow from the same computational solution and require only modest computing resources, making the approach practical in a resource-constrained setting.

VI. Discussion

The model developed here is, to our knowledge, the first mathematical description and numerical simulation of plastic spreading in Beira Lake. Its strengths are transparency and reproducibility: every step, from the governing diffusion equation through the finite-difference discretization and the von Neumann stability analysis to the verified second-order convergence and the computed lake-scale simulation, is derived from first principles and can be independently reproduced. The framework is also flexible and inexpensive, running on ordinary hardware with open-source tools. The no-flux boundary condition provides an explicit, physically interpretable mechanism for shoreline accumulation (Section V), rather than an assumed outcome.

Several limitations and challenges should be stated plainly. First, and most important, there are as yet no field measurements of plastic concentration in Beira Lake, so the simulations are numerically validated (against the exact solution) but not yet field-validated, and the parameter values, though physically reasonable, are uncalibrated; obtaining in-situ measurements is the principal challenge for turning this framework into a predictive tool. Second, the pure diffusion model neglects any wind- or inflow-driven surface current; where such a current is significant (large Péclet number), the model should be extended to the advection-diffusion-reaction equation, adding a transport term $\mathbf{v} \cdot \nabla u$ and, for stability, an upwind discretization of that term [12], [14], [16]. Third, the idealized rectangular geometry and constant coefficients simplify a complex, irregularly shaped lake, and the single-scalar treatment ignores the range of sizes and buoyancies of real plastic debris. Each limitation points to future work: field sampling to calibrate and validate the model, coupling to a hydrodynamic model on the true lake geometry, and a multi-class description of the plastic population. A further natural extension is to compare the finite-difference solution against a physics-informed neural-network solver of the same diffusion equation.

VII. Conclusion

This paper developed and numerically solved a computational-mathematics framework for the spreading of plastic in Beira Lake, Colombo, based on the two-dimensional diffusion equation. The explicit, implicit and Crank-Nicolson finite-difference schemes were derived and their stability established by the von Neumann method (the diffusion-number restriction $r \leq \frac{1}{4}$ for the explicit scheme and unconditional stability for the implicit schemes). The solver was verified against the analytical Gaussian heat-kernel solution through a grid-refinement study, which confirmed second-order spatial accuracy: the L^∞ error decreased from 4.33×10^{-5} at $h = 1/20$ to 6.75×10^{-7} at $h = 1/160$, an observed order $p \approx 2.00$. Applied to an idealized representation of the lake, the model produced computed simulations that reproduce the outward diffusive spreading of plastic from the canal source (peak quasi-steady concentration ≈ 1.72, decaying smoothly to ≈ 0.2 across the basin) and the accumulation of plastic along the enclosed no-flux shoreline, a behaviour shown to follow directly from the boundary condition and consistent, qualitatively, with the lake's documented pollution pattern. Although the absence of field data means the results are at present illustrative, the framework provides a transparent, reproducible, low-cost and numerically validated tool for predicting plastic-concentration patterns. Future work will focus on field sampling to calibrate and validate the model, extension to the advection-diffusion-reaction equation on the true lake geometry, a multi-class

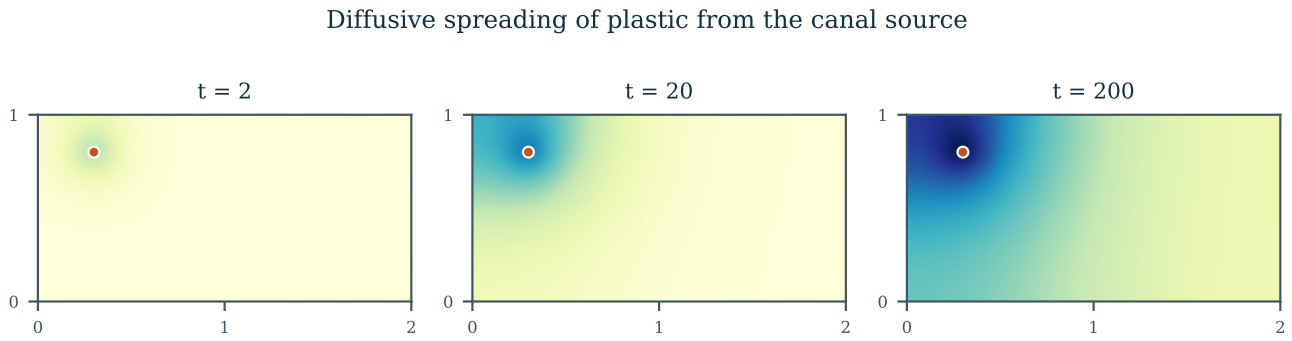


Fig. 2. Computed diffusive spreading of plastic from the canal source at three times, from the early transient ($t = 2$) through growth ($t = 20$) to the quasi-steady state ($t = 200$). The orange marker is the source; colour is plastic concentration on a common scale.

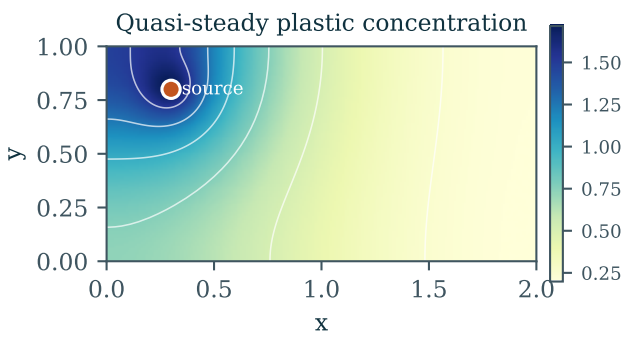


Fig. 3. Quasi-steady plastic concentration field on the idealized Beira Lake basin. White curves are concentration contours; the field is highest at the canal source and decays outward, with plastic banked against the enclosed upper-left shoreline.

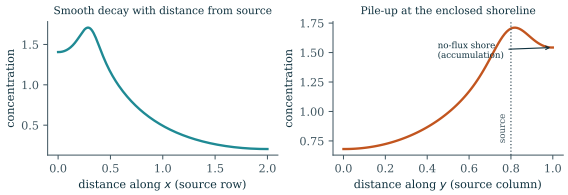


Fig. 4. (a) Computed concentration along the source row, showing smooth decay with distance from the source. (b) Computed concentration along the source column, showing the rise toward, and accumulation at, the enclosed no-flux shoreline.

plastic description, and a comparison against physics-informed neural-network solvers.

Acknowledgement

The authors used AI tools for brainstorming and language editing, and LaTeX for formatting.

References

[1] S. Prasad *et al.*, "Spatiotemporal assessment of water pollution for Beira Lake, Sri Lanka," *Water*, vol. 16, no. 11, art. 1616, 2024.

[2] D. Dharmarathna *et al.*, "Assessment of pollution state of Beira Lake in Sri Lanka using water quality index, trophic status, and principal component analysis," *Aquatic Ecol.*, vol. 58, pp. 159–174, 2024.

[3] V. P. A. Weerasinghe and K. Handapangoda, "Surface water quality analysis of an urban lake; East Beira, Colombo, Sri Lanka," *Environ. Nanotechnol. Monit. Manage.*, vol. 12, art. 100249, 2019.

[4] R. M. L. S. Bandara, M. D. D. Perera, P. I. A. Gomes, and X.-F. Yan, "Profiling microplastic pollution in surface water bodies in the most urbanized city of Sri Lanka and its suburbs," *Water Air Soil Pollut.*, vol. 234, art. 157, 2023.

[5] M. Sewwandi *et al.*, "Unprecedented marine microplastic contamination from the X-Press Pearl container vessel disaster," *Sci. Total Environ.*, vol. 828, art. 154374, 2022.

[6] R. R. M. K. P. Ranatunga and D. S. Wijetunge, "A comprehensive review of microplastic contamination in Sri Lanka: sources, sinks, and pathways," in *Combating Plastic Pollution in Terrestrial Environment*. Singapore: Springer, 2025.

[7] N. Portillo De Arbeloa and A. Marzadri, "Modeling the transport of microplastics along river networks," *Sci. Total Environ.*, vol. 911, art. 168227, 2024.

[8] N. Portillo De Arbeloa and A. Marzadri, "Modeling microplastic dynamics in riverine systems: fate and transport analysis," *Environ. Sci. Pollut. Res.*, 2025.

[9] G. Saha, A. K. Saha, and A. Bhowmik, "Analytical modeling of microplastic transport in rivers: incorporating sinking, removal, and multi-phase dynamics," *Pollutants*, vol. 6, no. 1, art. 18, 2026.

[10] A. Pilechi, A. Mohammadian, and E. Murphy, "A numerical framework for modeling fate and transport of microplastics in inland and coastal waters (CaMPSim-3D)," *Mar. Pollut. Bull.*, vol. 184, art. 114119, 2022.

[11] W. Hundsdorfer and J. G. Verwer, *Numerical Solution of Time-Dependent Advection–Diffusion–Reaction Equations*. Berlin: Springer, 2003.

[12] R. J. LeVeque, *Finite Volume Methods for Hyperbolic Problems*. Cambridge, U.K.: Cambridge Univ. Press, 2002.

[13] K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations*, 2nd ed. Cambridge, U.K.: Cambridge Univ. Press, 2005.

[14] S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*. New York: Hemisphere, 1980.

[15] L. S. Andallah and M. R. Khatun, "Numerical solution of advection-diffusion equation using finite difference schemes," *Bangladesh J. Sci. Ind. Res.*, vol. 55, no. 1, pp. 15–22, 2020.

[16] T. Saad and M. Karam, "Stable timestep formulas for high-order advection–diffusion and Navier–Stokes solvers," *Comput. Fluids*, vol. 244, art. 105564, 2022.

[17] M. Th. van Genuchten and W. J. Alves, *Analytical Solutions of the One-Dimensional Convective-Dispersive Solute Transport Equation*. Washington, DC: USDA Tech. Bull. 1661, 1982.